

# Chapter 1

## Numbers: Concepts and Properties: Session 1

Goal: Students understand prime numbers, positive/negative, factors and multiples, can solve multistep word problems.

So who likes memorizing equations, raise your hands? Well for those of you who raised their hand you must be the kind of people who like working with fractions, and enjoy playing trivial pursuit...weirdos. For the rest of us who hate memorizing formulas (myself included) I am going to try to keep the formulas down to a minimum. I'll try to share with you ways to logically figure out the answer for yourself, and if your clever enough you should be able to logically create the formula yourselves. In the real world if you ever had to look up a formula, I would recommend any Mathematical Handbook of Formulas and Tables that you find at pretty much any books store.

### 1.1 Number Basics

#### 1.1.1 "Well that's just PRIME"

A prime number is a number that can only be divided by the numbers 1 and itself. The first couple of prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97...

As a side note the largest known prime number was discovered in 2006 by Curtis cooper and Steven Boone, which is  $2^{32,582,657} - 1$  (I wouldn't want to write this number out because it has almost 10 million digits. For those of you potential mathematicians reading this (and I know some of you are...or should be), if you can find the first prime number that has at least 10 million digits you could win \$100,000, awarded by the Electronic Frontier Foundation.

### 1.1.2 the Prime Factor

What it means to break a number down to its primes is to rewrite the number so that you have only prime numbers being multiplied by each other. Instead of using the cunning elegant sniper method, I like to use the machine gun when doing this. What that means is I'm going to hit a number with something simple that works every time, a lot of times. If I have a number I simply divide out the number 2 as much as I can, then when I can't go any further with 2 I move on to 3, then 5, 7, 11 and so on. Usually by the time you get to 5 or 7 the number is completely broken down to all of its primes. For example if I were to break down a number like 80

$$\begin{aligned} &80 \\ &2 \times 40 \\ &2 \times 2 \times 20 \\ &2 \times 2 \times 2 \times 10 \\ &2 \times 2 \times 2 \times 2 \times 5 \\ &2^4 \times 5 \end{aligned}$$

Now let's try 420

$$\begin{aligned} &420 \\ &2 \times 210 \\ &2 \times 2 \times 105 \\ &2 \times 2 \times 3 \times 35 \\ &2 \times 2 \times 3 \times 5 \times 7 \\ &2^2 \times 3 \times 5 \times 7 \end{aligned}$$

### 1.1.3 Special Numbers

The world of math is full of special numbers.

$$\pi = 3.141592653589793238462643\dots$$

$$e = 2.718281828459045235360287\dots = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \text{natural base of logarithms}$$

$$\sqrt{2} = 1.41, \sqrt{3} = 1.73$$

$$\gamma = 0.577215664901532860606521\dots = \text{Euler's constant} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln n\right)$$

## 1.2 Expo $x^n$

### 1.2.1 Exponent Basics

Exponentiation is a mathematical operation involving a the base and the exponent (the little number floating on the right hand side). This is actually a fancy way of writing out a long line of multiplications. If  $n$  is a natural number,

then

$$x^n = x \cdot x \cdot x \cdot x \cdot x \cdots x$$

with  $n$  factors of  $x$

Well one question is does this work for any value  $n$ ? The answer is yes, but if the exponent is a fraction or a negative number then we have to do things a little differently.

### Negative Exponents

If the exponent  $n$  is negative then we simply turn this into a fraction.

$$\begin{aligned} x^{-n} &= \left(\frac{1}{x}\right)^n \\ \left(\frac{x}{y}\right)^{-n} &= \left(\frac{y}{x}\right)^n \end{aligned}$$

You aren't always going to want to deal with negative exponents so all you have to do is reciprocate the base (turn the big number upside down) and make the exponent positive.

### Dude that's Radical

If all expressions represent real numbers, then

If the exponent  $n$  is a fraction then were looking at the whole concept of a root. For example if we are talking about the square root of  $x$  we can kind of think of raising  $x$  to the  $\frac{1}{2}$  power,  $\sqrt{x} = x^{\frac{1}{2}}$ . The reverse of this also works,  $x^{\frac{1}{3}} = \sqrt[3]{x}$

$$\begin{aligned} x^{\frac{1}{n}} &= \sqrt[n]{x} \\ x^{\frac{m}{n}} &= \left(x^{\frac{1}{n}}\right)^m = \sqrt[n]{x^m} \end{aligned}$$

Heres a few other properties about Radicals

$$\begin{aligned} x^{\frac{1}{n}} &= \sqrt[n]{x} \\ x^{\frac{m}{n}} &= \sqrt[n]{x^m} \\ (\sqrt[n]{x})^n &= x \\ \sqrt[n]{x^n} &= |x| \text{ if } n \text{ is even} \\ \sqrt[n]{x^n} &= x \text{ if } n \text{ is odd} \end{aligned}$$

## 1.2.2 Work with squares and square roots of numbers

In the 16<sup>th</sup> century Robert Recorde got tired of saying “to the second power,” “to the thrid power,” and so on and so forth. So he came up with the terms square (second  $x^2$ ), cube (third power  $x^3$ ), zenzizencic (fourth power  $x^4$ , a.k.a. biquadrate), surfolide (fifth  $x^5$ ), zenzicube (sixth  $x^6$ ), second surfolide (seventh  $x^7$ ) and Zenzizenzizencic (eighth  $x^8$ ). Some where along the line people got tired of using all these crazy names and only kept the terms square and cube. In almost all fields of science this square or square root will pop up, so it would be best to familiarize yourself with this. Keep in mind that the square of a number (whether positive or negative), along with all other even exponents will always give you a positive number.

Square root of a number  $x$  is a number  $a$  such that  $a^2 = x$ , or in words, a number  $r$  whose square (the result of multiplying the number by itself) is  $x$ . The symbol that was chosen to represent square roots looks like some kind of funky check mark  $\sqrt{\quad}$ . As mentioned before the root comes in different degrees and we usually put the degree of the root just to the left of the  $\sqrt{\quad}$  symbol. Now after a while people started to realize the most common of all the roots was the  $\sqrt{\quad}$ , so eventually in stead of writing the 2 they just dropped it and the 2 is implied. That is, if there is no number in front of the root then assume that its 2. Keep in mind that you if the number under the square root is negative then that brings us to a slightly different creature called “complex numbers,” but don’t worry were going to hit that in the next section.

## 1.2.3 Work with cubes and cube roots of numbers

### 1.2.4 Apply rules of exponents

Ok so heres a bunch of formulas that I expect you to know; note that you don’t have to necessarily memorize them, but you will have to know how to figure them out. I’m going to use a few letters in this next section, so when you see either an  $m$  or an  $n$  just think of those as integer numbers, and  $x$  and  $y$  are going to be the base of my exponents. Lastly if you see a fraction we need to assume that the denominator (bottom) of that fraction is not zero, or else we would have an undefined number.

**Product Rule of Exponents**  $x^m x^n = x^{m+n}$ , and **Quotient Rule of Exponents**  $\frac{x^m}{x^n} = x^{m-n}$

If you came across a problem that said  $x^5 \cdot x^3 = ?$  and you didn’t know the trick the first thing I would do is write it all out so I can see what’s going on.

$$x^5 \cdot x^3 = (x \cdot x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^8 = x^{5+3}$$

If you do enough of these problems you start to realize that if the two terms are being multiplied and have the same base but different exponents then all you have to do is add the exponents.

$$x^m x^n = x^{m+n}$$

Notice how this only really works when the base for both terms are the same. If I had something like  $x^5 \cdot y^3 = ?$ , I could multiply it all out and discover that  $x^5 \cdot y^3 = x^5 \cdot y^3$ , thats like combining apples and oranges.

Heres the my math proof for the Quotient Rule, which is very similar to the Product Rule

$$\frac{x^m}{x^n} = x^m \left( \frac{1}{x^n} \right) = x^m x^{-n} = x^{m-n}$$

You need to make sure that the two numbers are the same base, and they both are not zero. An example of this would be

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x \cdot x = x^2 = x^{5-3}$$

Also keep in mind that this will work for fractional exponents as well.  $\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$  and  $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

**the Power Rules of Exponents**  $(x^m)^n = x^{mn}$ ,  $(xy)^n = x^n y^n$ ,  $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Ok so check out this  $(xy)^n = x^n y^n$  see how it really only works when everything is being multiplied. Make sure that you don't mix this up when we have an exponent over a sum  $(a+b)^n$ . One of the most common mistakes math kids make is they think they can just distribute the exponent to each term and make  $(a+b)^2 = a^2 + b^2$ , THIS IS ABSOLUTE WRONG.  $(a+b)^n$  is called binomial expansion and I'll teach you how to deal with that later.

**Zero's and Ones**  $x^0 = 1$ ,  $x^1 = x$ ,  $0^0 = \text{undefined or } 0^0 = 1$

Anything to the zeroth power is 1. To show you this we have to understand that any number divided by itself is the number 1, specifically  $\frac{x^n}{x^n} = 1$ .

$$1 = \frac{x^n}{x^n} = x^{n-n} = x^0$$

### 1.2.5 Rational Exponents

$$\begin{aligned} \left(x^{\frac{1}{n}}\right)^n &= x \\ x^{\frac{m}{n}} &= \left(x^{\frac{1}{n}}\right)^m = \left(x^m\right)^{\frac{1}{n}} \\ x^{-\frac{m}{n}} &= \frac{1}{x^{\frac{m}{n}}} \end{aligned}$$

1. Calculate or Simplify  $(3^2)^3$
2. Calculate or Simplify  $a^2 b^3 a^3 b^2$
3. Calculate or Simplify  $(x^3 y^4)^4$
4. Calculate or Simplify  $\left(\frac{a^{-2} b^3}{a^2 a^3 b^5}\right)^3$
5. Calculate or Simplify  $\left(\frac{2}{3}\right)^{-4}$
6. Calculate or Simplify  $\left(\frac{2x^{-2}}{3y^3}\right)^{-4}$



numbers like  $\frac{10^9}{10^{12}}$  you can keep in mind that a fraction simply means a negative exponents, so

$$\begin{aligned} 10^9 \div 10^{12} &= \frac{10^9}{10^{12}} \\ \frac{10^9}{10^{12}} &= \frac{10^9}{1} \times \frac{1}{10^{12}} \\ &= 10^9 \times 10^{-12} \\ &= 10^{9+(-12)} \\ &= 10^{-3} = .001 \end{aligned}$$

is that simple.

### 1.3 Imaginary Numbers $i = \sqrt{-1}$ , It's not really that Complex $z = a + bi$

The imaginary unit is denoted by the letter  $i$  which is defined as

$$i = \sqrt{-1}$$

So it follows that  $i^2 = -1$ . A Complex number is a number that has both a real part and an imaginary part and takes the form

$$z = a + bi$$

where  $a$  is the real part

and  $b$  is the imaginary part

#### 1.3.1 Complex Adding and Subtracting

Notice that the real part and imaginary parts of a complex number have to keep themselves separate. So when adding two complex numbers,  $a + bi$  and  $c + di$  you need to remember that you can only add the real part to the other real part and you can only add the imaginary part to the other imaginary part. The same goes for subtraction. In a sense you can treat  $i$  as a variable.

$$\begin{aligned} (a + bi) + (c + di) &= (a + c) + (b + d)i \\ (a + bi) - (c + di) &= (a - c) + (b - d)i \end{aligned}$$

#### 1.3.2 Complex Multiplication and Fractions

Have you ever heard of the FOIL method. It has to do with multiplying binomials, and it stands for First Outer Inner Last. So when you want to multiply something like  $a + bi$  with  $c + di$  you get

$$(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$$

In all honesty though we usually don't just multiply two complex numbers together. One of the things that we like doing is multiplying a complex number by its complex conjugate, what this does is it gets rid of the imaginary part. A complex conjugate is the complex number but the imaginary part has its sign switched,  $a + bi$  and  $a - bi$  are complex conjugates.

$$(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$$

This trick becomes very helpful when you are dealing fractions and you don't want to have imaginary numbers in the denominator. You simply multiply both the numerator and the denominator by the complex conjugate of the denominator and simplify.

$$\begin{aligned} \frac{6 + 3i}{2 - 4i} &= \frac{6 + 3i}{2 - 4i} \cdot \frac{2 + 4i}{2 + 4i} \\ &= \frac{12 + 24i + 6i - 12}{4 + 16} \\ &= \frac{30i}{20} \\ &= \frac{3i}{2} \end{aligned}$$

Speaking of imaginary numbers in the denominator notice that  $\frac{1}{i} = -i$

I suppose one of the most popular questions to be asked with the section on complex numbers is “why the heck is this important to me? When will we possibly use this in real world math.” Surprisingly complex numbers rear their heads in a vast array of fields including applications in engineering, electromagnetism, quantum physics, differential equations, and chaos theory, to name a few. I think one of the coolest things that happened to me in math is when I was in a math class called Partial Differential Equations. One of the problems looked absolutely impossible to do, and I'll never forget what my teacher said “Oh this problem is simple, all you have to do is make it complex.” Wait the words simple and complex should not be so close to each other in the same sentence. She continued to blow my mind by adding “well the complex plane is really just a shadow of the real plane.” Ok so here's what she did, she took the crazy differential equation and transformed it over to the complex plane. She then solved the answer in the complex plane using basic algebra skills. Transformed it back to the real plane and circled the answer. At that point I thought that complex numbers could possibly be one of the coolest things ever.

1. Add  $(7 - 4i) + (9 + 2i)$
2. Subtract  $(2 - 4i) - (-4 + 3i)$
3. Multiply  $(3 + i)(1 + 2i)$
4. Multiply  $(-4 + 2i)(2 + i)$
5. Find the product of the complex number  $3 + i$  and its complex conjugate.

6. Get rid of the imaginary number in the denominator of  $\frac{3-i}{2+i}$
7. Get rid of the imaginary number in the denominator of  $\frac{5+i}{5-i}$
8. Get rid of the imaginary number in the denominator of  $\frac{(2-5i)-(5-2i)}{(5+2i)+(2+i)}$

## 1.4 Logarithms

Logs are used everywhere with an incredible amount of applications. Some of the things that utilize logs range from pH scale, measuring earthquakes, astronomy, computer science, music, geometry, etc... A lot of people think that Logarithms are hard if you don't have a chart or a calculator. But actually logarithms aren't that hard once you get the hang of them. What we are really interested in is the definition of logarithms and also using the right properties. First of all the definition of a logarithm is

$$y = \log_b x \quad \text{iff} \quad x = b^y \quad (b > 0 \quad \text{and} \quad b \neq 1)$$

$$y = b^x \quad \text{and} \quad y = \log_b x \quad \text{are inverse functions.}$$

Some of the terminology that is used with the above equations is we call that little  $b$  on the bottom right hand corner of the log the base.

Here are some properties of logs that make things really cool. I'll prove a few of them to you and then I'll try to show you some applications of logs.

$$\begin{aligned} \log_b 1 &= 0 \\ \log_b b &= 1 \\ \log_b b^x &= x \\ b^{\log_b x} &= x \\ \log_b (MN) &= \log_b M + \log_b N \\ \log_b \left( \frac{M}{N} \right) &= \log_b M - \log_b N \\ \log_b M^p &= p \log_b M \\ \text{If } \log_b x &= \log_b y \quad \text{then } x = y \\ \log_b x &= \frac{\log_a x}{\log_a b} \quad \text{Change-of-base formula} \end{aligned}$$

Let me prove to you that  $\log_b 1 = 0$ , well first we start off with the definition and translate  $\log_b 1 = 0$  into  $1 = b^0$ . As I said earlier, anything raised to the zeroth power is 1 and so its easy to see that illustrated in  $b^0 = 1$  no matter what  $b$  is.

Believe it or not, logarithms can actually simplify tough calculations. Which is easier to do in your head multiply two numbers or add two numbers? What about dividing two numbers versus subtracting two numbers.? Well

according to the properties above using logarithms will actually turn your crazy operations into things a little more chewable. For example I was once asked to figure out what  $6^{137}$  without using a calculator. I tried everything but couldn't find a solution... until I turned to logs. Just as a note to you, I've found the most useful log properties to be  $\log_b M^p = p \log_b M$  where you can take an exponent and just drop it in front of your log, and  $\log_b(MN) = \log_b M + \log_b N$ .

$$\begin{aligned} 6^{137} &= ? \\ \log_{10} 6^{137} &= \log_{10} ? \\ 137 \log_{10}(6) &= \log_{10} ? \\ 137 \log_{10}(2 \cdot 3) &= \log_{10} ? \\ 137(\log_{10} 2 + \log_{10} 3) &= \log_{10} ? \end{aligned}$$

At that time I had  $\log_{10} 2 = .3010$  and  $\log_{10} 3 = .4771$  memorized. You don't have to memorize them, but for some reason I did.

$$\begin{aligned} 137(\log_{10} 2 + \log_{10} 3) &= \log_{10} ? \\ 137(.3010 + .4771) &= \log_{10} ? \\ 137(.7781) &= \log_{10} ? \\ 106.6 &= \log_{10} ? \\ 106 + 0.6 &= \log_{10} ? \\ 10^{106+0.6} &= 10^{\log_{10} ?} \\ 10^{0.6} \times 10^{106} &= ? \end{aligned}$$

And that's my answer. If I had a calculator I could calculate what  $10^{0.6}$  is, but more importantly I'm able to accurately tell you what the order of magnitude is on this sucker  $\times 10^{106}$ .

1. Figure out what  $x$  is if  $\log_3 81 = x$
2. Figure out what  $x$  is if  $\log_x 125 = 3$
3. Figure out what  $x$  is if  $\log_4 x = 3$
4. Figure out what  $x$  is if  $\log_{\frac{1}{3}} x = -2$
5. Simplify  $\log_7 7^3$
6. Use the properties of logarithms to show that  $\frac{1}{2} \log_b(x+2) - \log_b y + 3 \log_b z = \log_b \frac{z^3 \sqrt{x+2}}{y}$
7. Use the properties of logarithms to expand  $\log_b \frac{y^2 \sqrt{z}}{x}$

8. Knowing that  $\log_{10} 2 = 0.3010$  and  $\log_{10} 3 = 0.4771$  guess what  $\log_{10} \sqrt{5}$  is.
9. Knowing that  $\log_{10} 2 = 0.3010$  and  $\log_{10} 3 = 0.4771$  guess what  $\log_{10} \frac{3}{2}$  is.

## 1.5 Number Patterns

For the most part this can be thought of as kind of a puzzle or a game. I can't think of any extra clever way of teaching these to you besides just giving you a lot of problems to practice. What you have to learn to do is start to recognize the patterns. My best recommendation for these are to write them out, then start putting down any information that might help out. Here's a few tips about number patterns.

### 1.5.1 Arithmetic Sequences $a_n = a_1 + (n - 1)d$

an Arithmetic Sequence is made by adding some value to the sequence each time. For example the sequence 1, 4, 7, 10, 13, 16, 19, 22, 25, ... is made by adding the number 3 to the last number. In math language this sequence can be defined by

$$S_n = a_1, (a_1 + d), (a_1 + 2d), \dots + (a_1 + (n - 2)d), (a_1 + (n - 1)d)$$

### 1.5.2 Geometric Sequence $a_n = ar^{n-1}$

A Geometric Sequence is made by multiplying the previous number by some value each time to get the next number. For example, can you guess how this sequence is formed? 3, 9, 27, 81, 243, 729, 2187, ...

### 1.5.3 Special Sequences

#### Square Numbers

In this sequence you get the next number by squaring where it is in the pattern. One of the most common ones is

$$1, 4, 9, 16, 25, 36, 49, 64, 81, \dots$$

Notice how the number in the sequence depends on where the number is. What I mean is the second number is 2 squared ( $2^2 = 4$ ), or the seventh number is  $7^2 = 49$  and so on and so forth. This place holder is called the index, and often it is very helpful to lightly write those below your sequence.

Sequence	1,	4,	9,	16,	25,	36,	49,	64,	81,	...
Index	1	2	3	4	5	6	7	8	9	...

#### Cube Numbers

This sequence is just the cube of the index

$$1, 8, 27, 64, 125, 216, 343, 512, 729, \dots$$

## Fibonacci Numbers

Try to figure out the pattern of this sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \dots$$

this one is actually quite important. It is called the Fibonacci Sequence and it is made by adding the two numbers before it together. There are so many interesting things about the Fibonacci Sequence and I wish that I had enough time to really share it all with you.

One of the most interesting things that I've found is how often the Fibonacci numbers pop up in nature. When you look at different flowers you will notice that quite often the number of petals they contain is a Fibonacci number. Even the seed heads in large flowers like a sun flower use Fibonacci numbers to figure out something called optimal packing. Shells of snails and sea shells often grow in a way that can be closely approximated by Fibonacci Rectangles.

Another interesting thing that can be done with the Fibonacci Numbers is their close relation to the Golden Ratio. If you take any two successive Fibonacci Numbers and take their ratio (divide them) the number you get is very close to the Golden Ratio  $\varphi = 1.618034\dots$ . And something even more interesting is you can use the Golden Ratio to get a formula for calculating any Fibonacci Number.

$$x_n = \frac{\varphi^n - (1 - \varphi)^n}{\sqrt{5}}$$

Here's a longer list

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, \\ 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, \dots$$

Try this on for size. the human body has a lot of ratios that relate really well to the golden proportions. Make the following measurements and calculate the ratios.

1. Your height compared with the distance from the floor to your belly button.
  2. The distance from the floor to your belly button compared with the distance from the floor to your knee cap.
  3. The length of your arm from the shoulder compared with the distance from your fingertips to your elbow.
  4. The distance from your chin to the center of your eyes compared with the distance from your chin to the tip of your nose.
  5. The length of your index finger compared with the distance from your index fingertip to your big knuckle.
1. What are the missing numbers in the sequence  $\_, 4, \_, 16, 32, 64, 1128, 256, \dots$
  2. Find the next two numbers in the sequence  $3, 8, 13, 18, 23, 33, 38, \_, \_, \dots$
  3. Find the missing numbers in the sequence  $1, \_, 9, \_, 81, -243, 729, \dots$
  4. What is the next number  $1, 3, 6, 10, 15, 21, 28, 36, 45, \dots$

1. Simplify  $\frac{c^{12}c^5}{(c^5)^2}$
2. Simplify  $\left(\frac{k^{-3}}{k^{-4}}\right)^{-1}$
3. Simplify  $\frac{(6x^{-3}y^2)^3(-4x^{-3}y^4)^{-3}}{(-12x^{-3}y^3)^{-2}}$
4. Simplify  $\frac{a^0+b^0}{2(a+b)^0}$
5. If 12 vases cost \$18.00, what is the cost of 1 vase? A. \$0.67  
B. \$1.05  
C. \$1.33  
D. \$1.50  
E. \$1.60
6. Kaya ran  $1\frac{1}{2}$  miles on Monday and  $2\frac{1}{3}$  miles on Tuesday. what was the total distance, in miles, Kaya ran during those 2 days?  
A.  $3\frac{2}{15}$   
B.  $3\frac{3}{5}$   
C.  $3\frac{2}{3}$   
D.  $3\frac{7}{15}$   
E.  $3\frac{11}{15}$
7. If  $i^2 = -1$ , then  $(2 - 3i)^2 =$  A.  $-5 - 12i$   
B.  $4 + 6i$   
C.  $4 + 12i$   
D.  $5 + 12i$   
E.  $13 - 6i$
8.  $3x^2 \cdot 2x^2y \cdot 4x^2y$  is equivalent to:  
A.  $9x^7y^2$   
B.  $9x^{12}y^2$   
C.  $24x^7y^2$   
D.  $24x^{12}y$   
E.  $24x^{12}y^2$
9. A company rents moving vans for a rental fee of \$25.00 per day with an additional charge of \$0.30 per mile that the van is driven. Which of the following expressions represents the cost, in dollars, of renting a van for 1 day and driving it  $m$  miles? A.  $0.30m + 25$   
B.  $25m + 30$   
C.  $30m + 25$

- D.  $25.30m$   
E.  $55m$
10. The expression  $a[b + (c - d)]$  is equivalent to:  
A.  $ab + ac - ad$   
B.  $ab + ac + ad$   
C.  $ab + ac - d$   
D.  $ab + c + d$   
E.  $ab + c - d$
11. If  $x$  is a real number such that  $x^3 = 64$ , then  $x^2 + \sqrt{x} = ?$  A. 4  
B. 10  
C. 18  
D. 20  
E. 47
12. For all positive integers  $x, y$ , and  $z$ , which of the following expressions is equivalent to  $\frac{x}{y}$ ? A.  $\frac{x \cdot z}{y \cdot z}$   
B.  $\frac{x \cdot x}{y \cdot y}$   
C.  $\frac{y \cdot x}{x \cdot y}$   
D.  $\frac{x - z}{y - z}$   
E.  $\frac{x + z}{y + z}$
13. Which of the following statements is NOT true about the arithmetic sequence 17, 12, 7, 2, ...? A. The fifth term is -3  
B. the sum of the first 5 terms is 35  
C. The eighth term is -18  
D. The common difference of consecutive terms is -5  
E. The common ratio of consecutive terms is -5
14. If  $a = b + 2$ , then  $(b - a)^4 = ?$  A. -16  
B. -8  
C. 1  
D. 8  
E. 16
15. If the first term in a geometric sequence is 3, and if the third term is 48, what is the 11th term? A. 228  
B. 528  
C. 110,592  
D. 3,145,728  
E. 12,582,912

16. For all  $x$ ,  $2^x + 2^x + 2^x + 2^x = A$ .  $2^{x+2}$   
B.  $2^{x+4}$   
C.  $2^{3x}$   
D.  $2^{4x}$   
E.  $2^{5x}$
17. In the complex numbers, where  $i^2 = -1$ ,  $\frac{1}{1+i} \cdot \frac{1-i}{1-i} = ?$   
A.  $i - 1$   
B.  $1 + i$   
C.  $1 - i$   
D.  $\frac{1-i}{2}$   
E.  $\frac{1+i}{2}$
18. The normal amount of lead in a certain water supply is  $1.5 \times 10^{-5}$  milligrams per liter. Today, when the water was tested, the lead level found was exactly 100 times as great as the normal level, still well below the Environmental Protection Agency's action level. What concentration of lead, in milligrams per liter, was in the water tested today?  
A.  $1.5 \times 10^{-105}$   
B.  $1.5 \times 10^{-10}$   
C.  $1.5 \times 10^{-7}$   
D.  $1.5 \times 10^{-3}$   
E.  $1.5 \times 10^{-\frac{5}{2}}$
19. What is  $\log_3 \sqrt{3}$ ? A.  $\frac{1}{9}$   
B.  $\frac{1}{3}$   
C.  $\frac{1}{2}$   
D. 1  
E. 3
20. Which of the following is the set of all real numbers  $x$  such that  $x + 3 > x + 5$ ?  
A. The empty set  
B. The set containing all real numbers  
C. The set containing all negative real numbers  
D. The set containing all nonnegative real numbers  
E. The set containing only zero
21. What 2 numbers should be placed in the blanks below so that the difference between consecutive numbers is the same? 17, -, -, 41  
A. 23, 24  
B. 24, 34  
C. 25, 33  
D. 26, 35

E. 27, 31

22. How many prime numbers are there between 30 and 50?

- A. 4
- B. 5
- C. 6
- D. 7
- E. 8

23. If  $\log_a x = s$  and  $\log_a y = t$ , then  $\log_a (xy)^2 = ?$  A.  $2(s + t)$

- B.  $s + t$
- C.  $4st$
- D.  $2st$
- E.  $st$

24. which of the following statements describes the total number of dots in the first  $n$  rows of the triangular arrangement illustrated below

·            1st row  
 ···        2nd row  
 ·····      3rd row  
 ·······    4th row  
 ········· 5th row

A. This total is always equal to 25 regardless of the number of rows.

⋮

- B. This total is equal to twice the number of rows.
- C. This total is equal to 5 times the number of rows.
- D. This total is equal to the square of the number of rows.
- E. There is no consistent relationship between this total and the number of rows.