Properties of Exponents and Square Roots

9.1 Properties of exponents

We need to review some properties of exponents before we introduce our new topic. We recall that by , we mean the product of 5 x's. From this definition, we see that

,

,

and

From these examples we abstract the following properties of exponents:

The equalities hold for any positive integers m and n and any numbers x and y.

We also see from the definition that

and

So, we have in general that

if ,

and

if .

These equalities hold for any positive integers m and n and any nonzero number x. These two properties are not esthetically pleasing because they depend on the relative sizes of the exponents. We would like to extend the definition of the exponents so that the property holds regardless of the relative sizes of the exponents. Then, since

for any exponent m and any nonzero number x, we have to define to mean 1 whenever x is not zero. Moreover, since

we have to define to mean . We summarize the new definitions:

for any nonzero number x.

for any positive integer n and any nonzero number x.

With these definitions, the property

holds for any positive integers m and n regardless of the relative sizes of m and n and for all nonzero number x. In fact, it is not difficult to show that the properties of exponents listed above hold for all integers, including the negative exponents and for all nonzero numbers x and y. We summarize the properties of exponents (also called the laws of exponents":

(A)

(B)

(C)

(D)

These equalities hold for any integers m and n and for any nonzero numbers x and y.

There are other properties of exponents, but since we do not use them, we do not list them.

To get a concrete feeling for these properties, compute the following using a scientific calculator:

(a) (a')

(b) (b')

(c) (c')

(d) (d')

(e) (e')

(f) (f')

(g) (g')

Example: Simplify the following expressions:

(a)

(b)

(c)

(d)

Solution: (a)

(b)

= =

Or by Property (C)

= by Property (B)

=

(c) or

(d)

=

=

=

Your instructor may specify the form of your answer, like to leave the your final results all in positive exponents. Follow your instructor's instruction.

4.

Chapter 9

Exercises 9.1

Simplify the following expressions: (Assume that all letters in the expressions stand for some nonzero numbers.)

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

9.2 Scientific notation

In studying sciences, particularly astronomy, we encounter very small as well as very large numbers. Such numbers are usually represented in so-called **scientific notation**. The scientific notation is a system of writing a decimal number as the product of a number between 1 and 10 and a power of 10. For example, the number is written as , and the number is written as . (That is, place the decimal point right after the first nonzero digit and multiply the number by an appropriate power of 10.) Recall that when we multiply a number by a power of 10, we merely shift the decimal point. For example, when we multiply the number by , we shift the decimal point of five places to the right, so the number is . Also when we divide a number by a power of 10, we shift the decimal point of the number to the left by appropriate places. For example, means that we divide by , we shift the decimal point of six places to the left, so that the decimal representation of the number is .

When we multiply and using a scientific calculator, the calculator displays the number in the scientific notation. However, the calculator cannot display the number as . It displays the number in some other way depending on the brand of calculator you are using. **You must learn to interpret the display of your calculator. You cannot copy down the display on your calculator because it means a totally different thing.** We now give some examples of computations involving scientific notation.

Example: Convert the numbers into scientific notation and then compute the expressions:

(a)

(b)

(c)

(d)

Solution: (a)

=

=

=

(b)

=

=

=

(c)

=

=

=

(d)

=

=

=

=

We wish to make a few comments.

* The decimal part of the number in scientific notation is seldom an exact number, and so the result of your computations involving scientific notation should be rounded off to a reasonable length. The equality should be interpreted in the sense of approximate equality.

* Your instructor may instruct you to use a scientific calculator to do the computations once you convert the numbers into scientific notation. In that case, you must learn to enter the number in scientific notation into the calculator and perform the computations. You must pay close attention to your instructor's instruction.

Exercises 9.2

1. If the numbers are not already expressed in scientific notation, express them in scientific notation and them compute the expressions:

(a) (b)

(c) (d)

(e) (f)

(g) (h)

(i) (j)

(k) (l)

(m) (n)

2. "one light-year" is the distance that the light travels in one year. The speed of light is about meters per second.

(a) If 1 year consists of 365 days, approximately how many miles is 1 light-year? (1000 meters is equal to 0.621 mile.)

(b) If the distance from the Sun to the Earth is about meters, how long will it take for the light to reach the Earth? Express your answer in a reasonable unit.

3. The Earth goes around the Sun in approximately circular orbit with the Sun as the center of the circle. The approximate distance of the Earth from the Sun is meters.

(a) Compute the speed with which the Earth is going around the Sun. Give the speed in meters per second, taking one year to be 365 days.

(b) Express the speed of the Earth in miles per hour if 1000 meters is equal to 0.621 mile.

(c) Express the speed of the Earth in feet per second if 1 mile is equal to 5280 feet.

4. The radius of the Earth is about meters.

(a) How many miles is the radius of the Earth?

(b) What is the distance around the Earth (along the Equator)?

(c) If we travel along the Equator at the speed of 11 miles per hour, how long will it take to go around the Earth? Express your answer in a reasonable unit.

5. In describing the staggering amount of our national debt, which was about 1.3 trillion dollars at the time, President Reagan said that if it were possible to string out 10 dollars bills to the Moon, 1.3 trillion dollars worth of ten dollar bills would stretch to the Moon and back. If the length of the ten dollar bill is 15.7 centimeters (or 0.157 meters) and the distance from the Earth to the Moon is about meters, was it an accurate statement?

6. The thickness of a ten dollar bill is about meters. If 1.3 trillion dollars worth of ten dollar bills were stacked up in one pile, how high will the pile be? Express your answer in a reasonable unit. (1000 meters is equal to 0.621 mile.)

9.3 Properties of square roots

The **square root** of a (positive) number x is a number whose square is equal to the number x. The square root of x is denoted by the symbol . Square roots of positive numbers are in general not so nice numbers in the sense that they are unending decimal numbers. However, there are "nice" square roots, and some of them are given below:

Etc.

Before we state some properties of square roots, we would like to find the following square roots using a calculator:

(a) (a')

(b) (b')

(c) (c')

(d) (d')

We find that (a) and (a') give the same number 3, and (b) and (b') give the same number . These examples illustrate the general property that

(1) and for any positive number a.

We also find that (c) and (c') give the same number , and (d) and (d') give the same number . Note that 7 is , and . These two examples illustrate the property that

(2) for all positive numbers a and b.

The property illustrated in (d) and (d') used to be important in the olden days when square roots of numbers were found from a table. (Inexpensive calculators became available only in early 1980's.) The table usually listed approximate square roots of numbers between 1 and 100. So, if we wanted to find the (approximate) square of a number like 500, we had to bring down the number under the square root sign to below 100, like in the following:

The process of writing as is used to be called (and is still called) "simplification" of the radical expression. (The square root sign is also called **radical sign**.) The number under the radical sign is called **radicand**.

Example: Simplify the following radical expressions, (assuming that all letters stand for some positive numbers): (By the statement, we mean to reduce the radicands to as small numbers or expressions as possible.)

(a)

(b)

(c)

(d)

(e)

Solution: (a)

(b)

(c)

(d)

(e) We first simplify as . Then,

12.

Exercises 9.3

Simplify the following expressions: (Assume all letters stand for some positive numbers.)

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m)

(n)

(o)

(p)

(r)

(s)

(t)

(u)