Chapter 5

Fractional and Literal Equations

5.1 Fractional equations

The topics that we study in this chapter are not that new and in fact we have encountered some problems like the ones we will study. We will study how to solve equations that contain liberal amounts of fraction. Let us look at an example.

Example 1: Find the solution of the equation $\frac{5x+6}{3x-4}=7$.

Solution: The "obvious" thing to do is to "get rid of" the fraction on the left-hand side by multiplying both sides of the equation by the denominator of the fraction:

$$(3x-4)\cdot\frac{5x+6}{3x-4}=7(3x-4)$$

$$5x+6=21x-28$$

which is an equation that we can solve easily. We find the solution to be $x=\frac{33}{16}$, which we can verify by direct substitution into the left-hand side of the equation. (Use a calculator to do the computation.)

Let look at another example.

Example 2: Find the solution of the equation .

Solution: Again the "obvious" thing to do is to get red of the denominators by multiplying both sides of the equation by the denominators. We can do this either one at a time or multiply both sides by the product of the denominators, or by "cross multiplying". You should follow your instructor's suggestion. We will get rid of the denominators one at a time in our solution.

2.

Adding , which eliminates the "square terms", and combining similar terms, we obtain

which is an easy equation to solve. We get the solution .

When we have a more complicated equation with more than two fractions, the idea is to reduce the left-hand side to a single fraction and the right-hand side to a single fraction and apply the method of Example 2.

Example 3: Find the solution of the equation .

Solution: Since the right-hand side consists of a single fraction, we need to reduce the left-hand side to a single fraction. We will isolate the problem and simplify one expression inside the parentheses at a time:

=

=

=

=

So the equation becomes

Multiplying the fractions on the left-hand side, we have

This equation is not much different from the equation of Example 2, and using the same technique, we obtain the solution .

When the procedure is this involved, the solution should be checked using a calculator.

Exercises 5.1

1. Find the solution of each of the following equations:

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m)

(n)

(o)

(p)

2. Find the solution of each of the following equations.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

5.2 Literal equations

To get an idea of the topic, we begin with an example. The formula is used to convert the temperature given in Fahrenheit to the temperature in centigrade (or Celsius). For example, the room temperature is about Fahrenheit. Substituting 70 for F in the formula, we obtain

so that the room temperature is about 21.1 degree centigrade or simply .

On the other hand, suppose we want to convert some temperatures given in centigrade to temperatures in Fahrenheit. That is, suppose we want to find temperatures in Fahrenheit corresponding to the temperatures , , , and in centigrade. Of course, we can find the corresponding temperatures in Fahrenheit by solving each of the equations

But the procedures for solving all these equations are exactly the same, and so solving each of these equations separately becomes repetitious. So we treat C as a known number and solve the equation for F as follows. Multiply both sides of the equation by the reciprocal of to get rid of :

and so

Using this last equation, we can rapidly compute the corresponding temperatures in Fahrenheit. For example,

when , ;
when , ;
when , ;

when , .

In this way we can find the temperatures in Fahrenheit corresponding to the centigrade temperatures of , , , and .

In solving a more complicated equation involving several letters for one of the variables, that variable cannot appear on both sides of your final equation. Let us look at an example to illustrate this point.

Example: Solve the equation for y.

Solution: As in the case of solving fractional equations, we will get rid of the fractions first.

Now we have to isolate the variable for which we are solving:

Notice that in the final equation the "y" does not appear on the right-hand side.

Exercises 5.2

Solve each of the following equations for the indicated variable:

(a) for y.

(b) for x.

(c) for a.

(d) for h.

(e) for r.

(f) for b.

(g) for s.

(h) for x.

(i) for P.

(j) for y.

(k) for r.

(l) for v.

(m) for x.

(n) for r.

(o) for w.

(p) for u.

(q) for u.

(r) for x.

(s) for x.

(t) for x.