

2x2 Systems of Linear Equations

3.1 Algebraic solution of 2x2 systems

The problem here is to find the solution to a problem with two unknowns and two equations. Let us look at an example.

Example 1: Find two numbers x and y whose sum is 56 and whose difference is 38.

Solution: The first part and the second part of the sentence translates into the following two equations:

$$x + y = 56$$

$$x - y = 38$$

Notice that when we add these two equations, y cancels out and we obtain one equation consisting of only one unknown:

$$2x = 94$$

from which we find

$$x = \frac{94}{2} = 47$$

Substituting this value of x into one of the two equations given above, we obtain

So, the two numbers are 47 and 9.

We can check our result:

Thus, the two numbers do satisfy the conditions of the problem.

2.

In eliminating y in the procedure given above, we made use of the property of equality that if equals are added to equals, the results are equal. In solving a more complicated problem of this type, we may have to use the second property of equality that if equals are multiplied by equals, the results are equal. We will illustrate the use of these two properties of equality to solve a general problem. (Your instructor will give more examples with increasing complexity.)

Example 2: Find the solution of the system of equations

Solution: By the statement of the problem, we mean to find the values of x and y that satisfy **both** equations. (Such a problem is called 2 by 2 system, meaning that the problem consists of 2 equations in 2 unknowns.)

We have numbered the equations so that we can keep track of our computations easily.

The objective is to get rid of one of the unknowns so that we will have only one unknown. We will use the properties of equality to do so. Let us eliminate y . Then, we have to make the coefficients of y in both equations the same in magnitude but opposite in the signs. We can achieve this if we multiply the first equation by 2 and the second equation by 3. By multiplying the first equation by 2, we mean to multiply both sides of the first equation by 2, so that we get

or

Similarly, we multiply Equation (2) by 3:

or

We can condense the above computations in the form

Now when we add these equations, the y terms drop out and we get

We can obtain the value of y by substituting this value of x into Equation (1) or Equation (2) and solving the resulting equation. However, when the value of x is such a fraction, we can be more accurate to go through the elimination process, this time eliminating x. We abbreviate the process as follows:

So, the solution is .

Remember that **the solution consists of two numbers**, one for x and the other for y.

When the solution is so complicated, we should check:

Thus, the numbers and satisfy both equations, and so those two numbers constitute the solution of the system. Remember that **we must check both equations.**

4.

Exercises 3.1

In each case, find the solution of the system and check your solution:

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

3.2 Problems giving rise to 2x2 systems

One of the reasons why we have studied how to solve systems of equations is that in solving so-called word- problems, we can set up the equations more easily if we introduce more than one unknowns. Let us look at an example:

Example 1: A certain organization keeps its scholarship fund of \$150,000 in two accounts, Type A account which pays the interest at the rate of 3.5% per year and Type B account which pays the interest at the rate of 5.75% per year.

- (a) If the organization keeps \$70,000 in Type A account, what is the annual interest income from the two accounts?
- (b) If the organization needs the annual interest income of \$8,400 from the two accounts, how should \$150,000 be distributed into the two accounts so that the annual income from the two account is \$8,400?

Solution: (a) If \$70,000 is kept in Type A account, then the amount to be kept in Type B account is \$80,000. So,

$$\begin{aligned} \text{total annual interest income} &= 0.035(70,000) + \\ & 0.0575(80,000) \\ &= 7050 \text{ dollars} \end{aligned}$$

The annual interest income from the two accounts is \$7050.

- (b) Let x be the amount to be kept in Type A account, and y be the amount to be kept in Type B account. Then,

When we solve the system of equations (Solve the system yourself.), we obtain $x = 10,000$ and $y = 140,000$. So, \$10,000 is to be kept in Type A account and \$140,000 in the Type B account.

6.

Example 2: Tim was assigned the task of preparing hamburger meat for his company's picnic and was asked to prepare 150 pounds of hamburger meat. At the market where Tim shops, there are two kinds of hamburger meat - - lean hamburger meat with the fat content of 8% and the regular hamburger meat with the fat content of 25%.

- (a) If Tim buys 100 pounds of the lean hamburger meat and 50 pounds of the regular hamburger meat and mix them up, what is the fat content of his hamburger meat?

- (b) If Tim wants the fat content of his resulting hamburger meat to be 12%, how many pounds of the lean hamburger meat and how many pounds of the regular hamburger meat should he buy and mix them up?

Solution: (a) The fat content of the mixture is the weighted average of the fat contents of the components. So, fat content of his hamburger meat =
Therefore, the fat content of his hamburger meat is about 13.7%.

- (b) Let x = the amount of the lean hamburger meat, and y = the amount of the regular hamburger meat. Then,

Replacing the denominator of the second equation by 150, since , we can write the second equation as

Multiplying both sides of this equation by 150, we obtain

So, we have the system

When we solve the system (Solve the system yourself), we obtain and .

Therefore, he should buy about 114.7 pounds of the lean hamburger meat and about 35.3 pounds of the regular hamburger meat.

8.

Exercises 3.2

1. At a concert sponsored by a certain college, the admission was \$5 per person for students and \$8 per person for nonstudents.
 - (a) One evening, 175 students and 125 nonstudents attended a concert. Find the total receipt from the admission.
 - (b) On another evening, a total of 435 persons attended a concert and \$2436 was collected from the admission. Find how many students and how many nonstudents attended the concert.

2. In a multiple choice exam in a history class, for each correct answer 6 points were given and for each incorrect answer, one point was deducted..
 - (a) If a student answered 30 questions correctly and 14 questions incorrectly, what was his score?
 - (b) If another student answered 77 questions and received the score of 252 points, how many questions did he answer correctly?

3. At an annual party of a certain company, each employee paid \$15 and each guest paid \$25.
 - (a) If 46 company employees and 20 guests attended the party, what was the total receipt from the party?
 - (b) If a total of 130 persons attended the party and \$2380 was collected from the party, how many company employees and how many guests attended the party?

4. (a) To prepare for an office party, Marie was assigned the task of preparing hamburgers. She bought 10 pounds of lean hamburger meat at \$2.30 per pound and 40 pounds of regular hamburger meat at \$1.70 per pound and mixed them up. In effect, how much per pound did she pay?
 - (b) In the above problem, if the fat content of the lean hamburger meat was 15% and the fat content of the regular hamburger meat was 30%, what would be the fat content of Marie's resulting hamburger?

- (c) In Problem (b), if she wanted the fat content of the resulting hamburger to be 20% and wanted to prepare 50 pounds of hamburger, how much of each kind of hamburger meat should she buy?
- (d) In Problem (a), if she was given \$90 to prepare 50 pounds of hamburger, how much of each kind of the hamburger meat should she buy?
5. (a) If a student mixes 200 cc (cubic centimeters) of 10% hydrochloric acid solution with 600 cc of 40% hydrochloric acid solution, what is the concentration of the resulting solution?
- (b) If a second student wants to prepare 500 cc of 30% hydrochloric acid solution by mixing suitable amounts of 10% hydrochloric acid solution and 40% hydrochloric acid solution, how many cc's of the 10% solution and how many cc's of the 40% solution should he mix?
- (c) If a third student wants to prepare 150 cc of 20% hydrochloric acid solution by mixing suitable amounts of 40% hydrochloric acid solution and water, how many cc's of the 40% solution and how many cc's of water should he mix?
6. (a) A salt solution contained 7 grams of salt per liter of the solution. A student wanted to reduce the salt concentration of the solution, and so he added 5 liters of fresh water to 3 liters of the original solution. What was the salt concentration of the new solution?
- (b) If another student wants to prepare 15 liters of the solution whose salt concentration is 2.5 grams per liter, then how many liters of the solution whose salt concentration is 7 grams per liter and how many liters of water should he mix?
7. A certain organization has a scholarship fund of \$100,000, which it wants to keep in two accounts. Type A account pays the annual interest of 6.7% but the fund cannot be withdrawn easily and Type B account pays the annual interest of 3.2% but the fund can be withdrawn at any time. If the organization wants to collect \$5650

10.

from the annual interests of the two accounts, how much should be kept in the Type A account and how much should be kept in the Type B account?

8. At a recent football game, the students of the participating schools were admitted free while the students of the nonparticipating schools were charged \$1.50 per person and adults were charged \$5 per person for admission. If a total of 25,000 persons attended the game, and of that number, 9500 were students of the participating schools, and the total receipt from the admission was \$37,460. Find the number of the adults who attended the game.
9. A coffee company received an order for 300 pounds of a blend consisting of Kona coffee and Guatemalan coffee. The order further specified that the blend be priced at \$7.25 per pound. If Kona coffee sells for \$8 per pound and Guatemalan coffee for \$6 per pound, how many pounds of Kona coffee and how many pounds of Guatemalan coffee should be mixed to fill the order?
10. Machine A produces a certain item at the rate of 12 units per hour, and Machine B produces the same item at the rate of 18 units per hour. It costs \$50 per hour to operate Machine A and \$90 per hour to operate Machine B. It is desired to produce 138 units of the item and keep the production cost to \$655. Find the number of hours each machine is to be kept in operation.