Problems Leading to Linear Equations

2.1 Linear equations

Consider the problem: What is the number x which when multiplied by 8 gives 125?

We know at once that the number x must be greater than 10 since 10 times 8 is 80 but less than 20 since 20 times 8 is 160. We can narrow down our estimates and obtain a reasonable answer in a few minutes. However, we want a more efficient method of finding the answer. To do so, we formulate the problem into an equation:

$x(8) = 125$ or $8x = 125$

Since 8 times x is to be 125, we reason that x must be the number that is obtained by dividing 125 by 8. That is,

$$x = \frac{125}{8} = 15.625.$$

So, the number is $\frac{125}{8}$ or 15.625. The number $\frac{125}{8}$ or 15.625 is called **the solution of the equation** $8x = 125$.

To solve more complicated problems, we make use of the following **properties of equality**:

(i) If equals are added to equals, the results are equal.

(ii) If equals are multiplied by equals, the results are equal.

Since subtraction of a number is defined to be the addition of its negative, the statement, "If equals are subtracted from equals, the results are equal." is included in Statement (i). Similarly, since division by a number is defined to be the multiplication by its reciprocal, the property of equality about division is included in Statement (ii).

We can formally solve the simple equation given above as follows:

Of course, we do not write in such great detail, but it is important to know the principles being used.

Example: Find the solution of the equation .

Solution: We will write out the solution in great detail. Once you get used to, you can omit many of the steps.

The idea is to bring the equation to the form ax = b by making use of the properties of equality. To get rid of from the left-hand side, we add 37 to both sides of the equation:

or

To get rid of from the right-hand side, we add to both sides of this equation:

or

and so

Your instructor will give more examples and suggest what steps can be omitted, and will specify in what form the answer should be given.

Exercises 2.1

1. Find the solution of each of the following equations: (You should check your solution at least for some of the problems.)

(a) (b)

(c) (d)

(e) (f)

(g) (h)

(i) (j)

(k) (l)

(m)

(n)

(o)

(p)

(q)

2. (a) Find the number x that satisfies the equation .

(b) Evaluate the expression for the value of x that you found in (a).

(c) What do you conclude from your computation of (b)?

3. Suppose you have a wealthy grandmother and she tells you on the new year's eve that she will give you money under one of the two plans:

   Plan 1: She will give you $1 on the first Monday, $2 on the second Monday, $3 on the third Monday, $4 on the 4th Monday,

$5 on the 5th Monday, and so on until the end of the year ($52 on the 52nd Monday).

Plan 2: She will give you $1 on the first month, $2 on the second month, $4 on the third month, $8 on the fourth month, $16 on the 5th month, and so on, each month doubling the amount given in the preceding month, until the end of the year.

(a) Which plan is advantageous to you? (You need a calculator to compute the amount in Plan 2.)

(b) Instead of starting with $1 in Plan 1, you request that she starts with x dollars and each week she increases the amount by $1, so that she will give you dollars on the second Monday, dollars on the 3rd Monday, dollars on the 4th Monday, dollars on the 5th Monday, and so on, until the end of the year (that is, until the 52nd Monday). If you want to pay next year's tuition, which you estimate to be $5000, with the money, what should be the first amount x dollars?

(c) Suppose you request your grandmother to modify Plan 2 and begin with y dollars and double the amount she gives in the succeeding month, so that she gives dollars in the second month, or dollars in the 3rd month, or dollars on the 4th month, or dollars on the 5th month, and so on until the end of the year. If you want to accumulate $5000 by the end of the year, what should be the first amount y dollars?

2.2 Problems involving rates

We review the computational procedures of quantities that we commonly encounter. One of the quantities that we encounter is rate.

Let us look at one of the most commonly used rates --- the gasoline price. At present the price of premium gasoline is about $1.73 per gallon. Notice the word "per" in "per gallon". Almost all the rates are expressed in this or similar fashion. What this means in this case is that 1 gallon of the premium gasoline costs $1.73; 2 gallons of the premium gasoline costs 2 times $1.73 or $3.46; 3 gallons of the premium gasoline costs 3 times $1.73 or 3(1.73) dollars; and so on. In general, n gallons of the premium gasoline costs n times $1.73 or n(1.73) dollars or 1.73n dollars. In this case, we do not have to buy an exact multiple of a gallon.; we can buy an amount that can exceed an exact multiple of a gallon by any fractional amount, and we pay "proportionately" for the fractional amount. By this we mean that if the fractional part is one-half of a gallon, then we pay one-half of $1.73; if the fractional part is one-fourth of a gallon, then we pay one-fourth of $1.73; if the fractional part is two-thirds of a gallon, we then pay two-thirds of $1.73; and so on. For example, suppose we buy 12.4 gallons of the premium gasoline. This means 12 full gallons plus four-tenths of a gallon. So, the amount we pay is 12 times $1.73 plus four-tenths of $1.73, or in symbol

$$12(1.73) + 0.4(1.73) \text{ dollars.}$$

This can be written (by applying the distributive law in the other direction) as

$$(12 + 0.4)(1.73) \text{ dollars}$$

or

$$12.4(1.73) \text{ dollars.}$$

That is, the formula for the cost C of n gallons of the premium gasoline is

$$C = n(1.73)$$

regardless whether n is or is not a whole number. In this case we say that the same rate applies to the fractional part. This is the case in a great majority of cases and it can be assumed unless otherwise stated explicitly or is obvious from the context.

Another rate that we commonly encounter is the **concentration** of a solution. The concentration of a solution is the amount of the

particular substance per unit volume or per unit weight of the solution. It is given in a variety of ways. For example, the concentration of the salt in sea water is about 3.5 grams per liter. This means that every liter (which is about a quart) of sea water contains about 3.5 grams of salt. Since 1 cubic centimeter of sea water weighs about 1 gram, and there are 1000 cubic centimeters in 1 liter, 1 liter of sea water weighs about 1000 grams. So, the above statement can also be stated as "3.5 grams of salt per 1000 grams of sea water" or "0.35 grams of salt per 100 grams of sea water". From the last statement we can also say that the concentration of salt in sea water is about 0.35%.

The general practice of giving concentrations is as follows. If a liquid is dissolved in a liquid, it is a usual practice to express the concentration in percent by volume. Thus, for example, 15% nitric acid solution means that 15% of the solution is (pure) nitric acid (the rest being water); that is, 100 cubic centimeters of 15% nitric acid solution contains 15 cubic centimeters of pure nitric acid and 85 cubic centimeters of water.

On the other hand, if a solid is dissolved in a liquid, the concentration is usually given as so many weight of the solid per unit volume of the solution, just as the way the concentration of salt in sea water was given above.

If a solid is contained in a solid, the concentration is usually expressed in percent by weight. Thus, for example, on a tray of hamburger meat, we see the specification like "Fat content: 18%", meaning that 18% of the hamburger meat (by weight), or about one-fifth of the meat, is fat.

We now give further examples of the use of rates.

Example 1: A bottle of hydrogen peroxide solution says "3% hydrogen peroxide". If the bottle contains 16 fluid ounces, how much of that amount is the pure hydrogen peroxide?

Solution: The concentration of 3% hydrogen peroxide means that 3% of the amount (16 fluid ounces) is pure hydrogen peroxide. So,

Amount of hydrogen peroxide = = 0.03(16)

= 0.48

So, the amount of pure hydrogen peroxide is 0.48 fluid ounce or about of an ounce.

Example 2: We know that 1 mile consists of 5280 feet and that 1 hour consists of 60 minutes and each minute consists of 60 seconds, so that 1 hour consists of 60(60) or 3600 seconds. We want to know that if a car is traveling at 50 miles per hour, how many feet it travels in 1 second.

Solution: Since 1 mile consists of 5280 feet, 50 miles consists of 50 times 5280 or 50(5280) or 264,000 feet. So the statement "50 miles per hour" is the same as "264,000 feet per hour" or "264,000 feet per 3600 seconds". That is, the car travels 264,000 feet in 3600 seconds. So, in 1 second, the car travels of 264,000 feet, or (264,000) feet, or about 73.3 feet.

Example 3: In the Federal and State income tax return form, we find the statement of the form, "You can deduct medical expenses only the amount over 7.5% of your gross income" (to compute the taxable income).

(a) If your gross income were $30,000 and the medical expenses were $1,500, how much could you deduct?

(b) If your gross income were $30,000 and the medical expenses were $3000, how much could you deduct?

Solution: (a) 7.5% of $30,000 is = 0.075(30,000) = 2250 dollars. Since the medical expenses $1,500 were less than $2,500 , you could not deduct any amount.

(b) Your medical expenses $3000 exceeded $3000 by $750, and so you could deduct $750.

Example 4: A tile wall of length 72 feet and 6 layers high is to be constructed. The length of a tile is one and a half feet (18 inches).

(a) Compute the number of tiles needed to construct the wall.

(b) Compute the cost of the tiles to make the wall if each tile costs $1.20.

Solution: (a) Since the length of each tile is 1.5 feet, the length of n tiles (placed end-to-end) will be 1.5 times n feet and we want this to be equal to 72 feet. That is,

$1.5n = 72$

The solution of this equation is

So 48 tiles are required to construct one layer. Since the wall is to have 6 layers, the total number of tiles is 6 times 48, 6(48), or 288. Therefore, at least 288 tiles are required to construct the wall.

(b) Since each tile costs $1.20,

The cost of the tiles = 1.20(288) = 345.60 dollars

So, the tiles alone will cost at least $345.60 to make the wall.

Incidentally, all the computations in the above example are done using a calculator so as not to interrupt the chain of reasoning. We will continue to do so unless the numbers are so nice that we can do computations mentally. Furthermore, we should check the reasonableness of our answers.

Example 5: If the salt concentration of sea water is 0.0292 pound per gallon, how many gallons of sea water do we have to boil off in order to extract 10 pounds of salt?

Solution: Let x be the number of gallons of sea water to be boiled off. Then the amount of salt in the x gallons is 0.0292 times x pounds and we want this to be equal to 10 pounds. Therefore, we have to solve the equation

$0.0292x = 10$

The solution is x = . Using a calculator, we find that x = 342.46575.

So we have to boil off about 342 gallons of sea water to extract 10 pounds of salt.

Example 6: The car insurance renewal notice of a car owner says that the semi-annual premium is $659, and a note says that 15% merit discount has been applied. What would have been the semi-annual premium without the merit discount?

Solution: If we denote the semi-annual premium without the merit discount by R, then $659 is the result of 15% of R deducted from R. That is,

$$R - 0.15R = 659$$

We note that an expression of the type $2R + 3R$ can be combined to 5R by applying the distributive property:

$$2R + 3R = (2 + 3)R = 5R$$

Similarly, we can combine $R - 0.15R$ by applying the distributive property, noting that R is the same as 1R:

$$R - 0.15 = 1R - 0.15R$$

$$= (1 - 0.15)R$$

$$= 0.85R$$

So the above equation becomes

$$0.85R = 659$$

and the solution of the equation is

$$= 775.29412$$

Therefore, the semi-annual premium without the merit discount would have been $775.29 .

Exercises 2.2

In each case, state the computational procedure in one line and then compute the quantity or translate the statement into an equation and then solve the equation. Then, state your answer so that it sounds sensible. (Do not just write down a number and box it or circle it.)

1. Suppose the gas mileage of your car is 28 miles per gallon, and the full-tank of your car contains 12 gallons.

(a) How far can you go with the full-tank of gas?

(b) If the price of the gas is $1.75 per gallon, how much (in dollars) does the full-tank of gas cost?

(c) If you drive 200 miles per week, how many miles do you drive in one year? (Assume you drive all 52 weeks per year.) How much do you spend on the gas, if the price of the gas is $1.75 per gallon?

(d) If you want to keep the cost of driving to $80 per week, how many miles per week can you drive if the price of the gas is $1.75 per gallon?

2. The rating of a water pump says that it can pump water at the rate of 500 gallons per minute.

(a) In 20 minutes how many gallons of water can it pump?

(b) If a swimming pool can contain 22,400 gallons, how many minutes will it take the pump to fill up the swimming pool?

3. It is often necessary to convert the concentration given in one system to another system to get a better picture. Take for example the salt concentration of sea water, which was given as 3.5 grams of salt per liter of sea water.

(a) Express the concentration of sea water in pounds per gallon, given that

1 gram = 0.0022046 pound

1 liter = 0.264175 gallon.

(b) Suppose we want to set up a 70-gallon marine aquarium. In order to make the aquarium water as close to sea water as possible, how many pounds of salt must be dissolved in 70 gallons of fresh water to make the aquarium water?

4. In each case, translate the problem into an equation and then find the answer.

(a) Four times a certain number x is 56. What is the number x?

(b) One-half of a certain number y is 36. What is the number y?

(c) of a certain number u is 72. What is the number u?

(d) One and a half times a certain number w is 48. What is the number w?

(e) Two and a half times a certain number t is 75. What is the number t?

(f) Someone says that 60 is of another number x. What is the number x?

(g) 30% of a certain number z is 45. What is the number z?

(i) The number 85.4 was obtained by adding 25% of a number y to the number y. Find the number y.

5. Suppose that your car's gas mileage is 25 miles per gallon, and you want to take a trip what will involve driving about 800 miles.

(a) Find the approximate amount of gas you will use for the trip.

(b) If the price of gas is $1.45 per gallon, find the approximate cost of the gas for the trip.

6. Suppose that you want to drive from Los Angeles to San Francisco, a distance of about 480 miles. If you can drive at a speed of 55 miles per hour, how many hours will it take you to complete the trip?

7. You want to paint your house and you estimate that you have about 1800 square feet of walls to paint.

(a) If a gallon of paint can cover 400 square feet of walls, how many gallons of paint do you need?

(b) If a gallon of paint costs 28 dollars, how much should you allocate for the cost of the paint?

8. Suppose that we can drive a car at the constant speed of 50 miles per hour.

(a) How many miles do we travel in 7.5 hours?

(b) How long will it take us to cover the distance of 490 miles? Express your answer as so many hours and so many minutes (that is, convert the fractional hour to minutes).

(c) How many feet will it travel in 2 seconds? (1 mile is 5280 feet.)

9. When a certain store bought a certain item for $300, it marked up the price by 75%; that is, the store put the price that is $300 plus 75% of $300.

(a) Compute the selling price of the item.

(b) The store found that the item would not sell at the price computed in (a). So, it decided to put the item on sale and reduced the price by 45%. Find the sale price of the item.

(c) If the store sold the item at the sale price computed in (b), how much profit did the store make on the item? ("profit" in this case means the price at which the item was sold minus the purchase price.)

(d) If the store marked up the price of a TV by 75% and the TV is being sold at $427, how much did the store pay for the TV?

(e) If the TV in (d) is sold at $427, how much profit does the store make?

10. The owner of a service station knows that he marked up the price of every item by 40%. During a clearance sale, he reduces the price of every item by 20%.

(a) If the regular price of a tire is $60, what is the sale price of the tire?

(b) How much did the owner of the service station pay for the tire whose regular price is $60?

(c) How much profit does the owner of the service station make if he sells the tire at the price computed in (a)?

11. In a certain theatre, there are 500 general admission seats and 300 preferred seats. The preferred seats are twice as expensive as the general admission seats.

(a) In a performance at the theatre, the price of the general admission seat was $20. What was the total receipt from the admission if all the seats were sold out?

(b) In a special performance at the theatre, the estimate of the expenses is $80,000. How should the general admission seat be priced in order to cover the expenses? (Remember that each of the preferred seat is twice as expensive as the general admission seat, and assume that all seats will be sold out.)

(c) The director of the theatre feels that the price as computed in (b) is too steep. He feels that he can get "corporate sponsors" who will contribute to cover 30% of the cost. Then, how should the seats (the general admission seats and the preferred seats) be priced to cover the expenses?

12. In Hawaii we have so-called "excise tax", and the businesses must pay 4% of their gross revenue for the excise tax. However, the businesses can pass on the entire amount to the consumers. Suppose, for example, a store sells an item for $750. If the store collects 4% of $750 or $30, the total that the customer pays is $780. However, the store pays 4% of the total $780, so that it pays $31.20, and so the store loses for the excise tax. Consequently, businesses usually collect more than 4% of the amount of purchase. Here are the problems:

(a) If a store sells an item for $750, how much does the store collect for the excise tax so that the amount it collects is 4% of the total amount?

(b) The amount you computed in (a) is what percent of $750?

(c) Suppose that the store sells another item for $930. How much does the store have to collect for the excise tax so that 4% of the total amount is the amount collected for the excise tax?

(d) The amount collected for the excise tax is what percent of $930?

2.3 Averages and weighted averages

We all know what the average of a collection of numbers is and how to compute it for a particular case. For example, we are all familiar with the following use of the average: When a student scored 90, 85, 92, 87, and 98 in tests in a certain course during a semester, we compute his average as

his average score =

and take this number as a measure of his achievement in the course during the semester. Such use of the average to represent a collection of data is extremely common. However, we may not be fully aware of how pervasive the use of averages is. For example, the use of averages is assumed in such a statement as, "If the price of gas is $1.45 per gallon and your car's gas mileage is 25 miles per gallon, compute the cost of the gas to drive 800 miles." Because such quantities as the gas price and gas mileage are not constant, we have to use some numbers to represent those quantities, and the quantities that are often used are the averages. Then, there are quantities that behave very much like averages although they are rarely regarded as averages.

The average that we more frequently encounter is the "weighted average". To see how the concept arises, we look at an extreme case.

Example 1: If a student had the test scores of 70, 100, 100, 100, and 100, what is his average?

Solution: Of course, his average is the sum of the scores divided by 5. But to show a certain pattern, we write the computational procedure as follows:

His average score =

The numerator is the sum 70 + 100 + 100 + 100 + 100 , which we wrote as 1(70) + 4(100). We put 1 in front of 70 to emphasize that there is only one 70. The "coefficients" of the 70 and 100, that is, the numbers in front of 70 and 100, indicate how many 70's and how many 100's there are in the collection. Note that the denominator is the sum of the coefficients. Note also that the student received only two kinds of scores, namely 70 and 100, and that the average computed above is not the same as the simple average of 70 and 100, which is 85. To distinguish the above computed average (which is the true average)

from the second "simple average of 70 and 100", we often call the above computed average **"the weighted average"**, the weights for 70 and 100 being 1 and 4, respectively.

The situation like the one given above seldom occurs and may seem artificial. However, the situation like the example below does occur and gives rise to the same computational procedure.

Example 2: A teacher of a certain class said that there would be one "mid-term" exam and the final exam. The final exam, being comprehensive, would count as 4 exams. If a student scores 70 in the mid-term exam and 100 in the final exam, what is his average score for the class?

Solution: The computational procedure for his average score is exactly the same as in Example 1, and he has exactly the same average score, namely

His average score =

The teacher of Example 2 could also have said that the mid-term exam would count 20% and the final exam would count 80%. Then, the computational procedure for the same student would have been

His average score =

which is the same as the average computed in Example 2.

Weighted averages are also used in the following situation. Instead of keeping track of each score, we can compute the average and keep track of the number of scores. Let us look at an example.

Example 3: Suppose that a student scored 70, 80, and 96 in the first three tests in a course, so that his average up to the third exam is . In the next test, he scored 90. Can we compute his average for the four tests knowing only that his average for the first three tests is 82?

Solution: The answer is yes. Because of the way we compute the average, we can compute the sum of the first three test scores by knowing the average, namely by multiplying the average by 3. That is,

$$70 + 80 + 96 = 3(82)$$

So,

His average for the 4 tests =

It is more illuminating to write the above computational procedure as

His average for the 4 tests =

because then we see exactly what is happening. We see that when we use the average of the first three tests to compute the average of the four tests, we are putting the weight of 3 on the average of the first 3 test scores, which makes sense since the average for the first 3 test scores represents the 3 test scores.

The preceding type of the use of averages is very common when large numbers are involved. We will look at some examples.

Almost all financial institutions, such as the banks, savings and loan companies, credit unions, and credit card issuing companies, keep the "average daily balances" of each customer, and pay or charge interests based on the average daily balances. So, what are the average daily balances? Let us look at an example.

Example 4: The account of a certain person contained the following amounts at the end of each day during the first 10 days of a month:

1st day:	$2,800
2nd day:	$2,800
3rd day:	$3,600
4th day:	$3,600
5th day:	$3,600
6th day:	$3,600
7th day:	$2,500
8th day:	$2.500
9th day:	$2,500
10th day:	$2,500

(a) What is the average daily balance for the first 2 days?

(b) What is the average daily balance for the first 5 days?

(c) What is the average daily balance for the first 6 days?

(d) What is the average daily balance for the first 10 days?

Solution: (a) Of course, the average daily balance for the first 2 days is $2,800, since the amount has not changed. It can, of course, be computed as

Average daily balance for the 1st 2 days =

(b) Average daily balance for the 1st 5 days =

= 3,280 dollars

(c) We can compute the average daily balance for the first 6 days by applying the definition as we have done above in (a) and (b), or we can make use of the result of (b), noting that the result of (b) represents the daily balances of 5 days. We will compute it in both ways:

Method 1:
Average daily balance for the 1st 6 days =

= 3,333.33 dollars

Method 2:

Average daily balance for the 1st 6 days =

= 3,333.33 dollars

In any case, we obtain the average daily balance for the first 6 days to be $3,333.33 .

(d) We can again either use the definition or the result of (c). We will use the definition to compute the average daily balance and leave the computation by using the result of (c) for you to do:

Average daily balance for the 1st 10 days =

= 3000 dollars

Example 5: Suppose that the amounts given in Example 4 were the outstanding balances of the person's charge card that charges 18% per year, the interest being computed daily on the daily balance at the end of each day. The statement that the interest rate is 18% per year means that if $2800 is left unpaid for 1 year, then the amount of interest on $2800 is 0.18(2800) dollars. One day is of 1 year, and so the amount of interest on 1 day is of 0.18(2800) dollars or (0.18)(2800) dollars. Since the outstanding balance for the second day is the same, the amount of interest charged for the second day is the same, and so

Interest for the 1st 2 days = 2( )(0.18)(2800)

= 2.76 dollars

Similarly,

Interest for the 1st 3 days = 2( )(0.18)(2800) + (0.18)(3600)

= 4.54 dollars

Interest for the 1st 6 days = 2( )(0.18)(2800) + 4( ) (0.18)(3600)

= 9.86 dollars

Now let us compute the amount of interest for the first 6 days using the average daily balance. Since the average daily balance for the first 6 days is $3,333.33 , which represents the daily balances of the first 6 days, we suspect that the interest for the first 6 days is

6( )(0.18)(3,333.33)

which when computed gives 9.86 dollars, the same result as above. The question naturally arises whether these two methods always give the same result, and if so why.

(a) Compute the amount of interest for the first 10 days using the daily balance.

(b) Compute the amount of interest for the first 10 days using the average daily balance.

(c) Explain why the two methods give the same result, at least for this example.

Solution: (a) Interest for the 1st 10 days

= 2( )(0.18)(2800) + 4( )(0.18)(3600) + 4( )(0.18)(2500)

= 14.79 dollars

(b) Interest for the 1st 10 days = 10( )(0.18)(3000)

= 14.79 dollars

(c) Noting that we have the common factor of (0.18), we apply the distributive property in the other direction, and we transform the expression of (a) as follows:

2( )(0.18)(2800) + 4( )(0.18)(3600) + 4( )(0.18)(2500)

= (0.18)[2(2800) + 4(3600) + 4(2500)]

=

= (0.18)(10)(3000)

which is the expression by which we computed the interest for the first 10 days using the average daily balance. Thus, we see why the two methods give the same result. Moreover, we see that the same argument will work to show that the two methods always give the same result since our argument does not depend on the particular numbers we used.

This result shows the reason why banks and other financial institutions compute the average daily balance for each month and use it to compute the interest at the end of each month.

As a final example, we give a quantity that behaves like an average.

Example 6: If we mix 10 pounds of hamburger meat whose fat content is 15% with 40 pounds of hamburger meat

whose fat content is 30%, what is the fat content of the mixture?

Solution: The amount of fat in 10 pounds of the hamburger meat with 15% fat content is 0.15(10) pounds, and the amount of fat in the 40 pounds of the hamburger meat with 30% fat content is 0.3(40) pounds. So, the amount of fat in the mixture is 0.15(10) + 0.3(40) or 13.5 pounds. The total amount of the hamburger meat is 10 + 40 or 50 pounds. So, the amount of fat in 1 pound of the mixture is of 13.5 pound. That is,

Fat content of the mixture = (13.5) = 0.27

So, the fat content of the mixture is 0.27 pound of fat per pound of the mixture, or 27%.

It is much more illuminating to write the computational procedure without simplifying and write the numerator as 10(0.15) + 40(0.3) as follows:

Fat content of the mixture =

Thus, we see that **the fat content of the mixture is the weighted average of the fat contents of the components**.

Since the computations involving concentrations of solutions are similar, when two or more solutions of different concentrations are mixed, **the concentration of the resulting solution is the weighted average of the component solutions,** the weights being the respective amounts of the component solutions.

Exercises 2.3

In doing each of the following problems, make sure to write down the computational procedure and observe the pattern. Then, for those problems that cannot be solved directly, set up the equations, solve the equations, and state your answers clearly.

1. In a math class, Tom received the scores of 75, 80, 84, 88 , and 95 for the first 5 quizzes.

(a) What is the average quiz score for the first five quizzes?

(b) His average for the next 3 quizzes was 92.6. What is his new quiz average (for the eight quizzes)?

(c) He wants to raise his quiz average to 90 and there are 4 more quizzes left. What must be the average of the remaining 4 quizzes in order to raise his quiz average to 90?

(d) Another student has the quiz average of 85 with 8 quizzes taken and wants to raise his quiz average to 92. If he expects to have 4 more quizzes, what should be the average of the 4 quizzes in order to raise his average to 92?

(e) A student has the quiz average of 85 with 8 quizzes taken and wants to raise his average to 92. If he can average 97 on the rest of the quizzes, how many quizzes does he need to raise his average to 92?

2. A standardized test was given to four algebra classes with the following results:

The first class had 25 students and the class average of 85.6;
the second class had 30 students and the class average of 81.2;
the third class had 20 students with the class average of 76.8;
the fourth class had 15 students with the class average of 90.0.

What was the average score in the standardized test for the four classes combined?

3. To determine the average price of the premium gasoline in Honolulu, a survey was made with the following results. There were

8 service stations selling the gas at $1.50 per gallon;
20 service stations selling the gas at $1.55 per gallon;
2 service stations selling the gas at $1.60 per gallon.

(a) What was the average price of the premium gasoline in Honolulu at the time the survey was taken?

(b) From the past experience, Jill knows that she drives her car about 500 miles in one month and that her car's gas mileage is about 25 miles per gallon. On the basis of the survey and above data, what will be her expected monthly expense for the gasoline for her car?

4. Shown below are the daily balances of the checking account of a person for the first 10 days of the month:

June 1: $4000

June 2: $3500
June 3: $3500
June 4: $3500

June 5: $3000
June 6: $3000

June 7: $2400
June 8: $2400
June 9: $2400
June 10: $2400

(a) What is the average daily balance for the first 2 days?

(b) What is the average daily balance for the first 4 days?

(c) What is the average daily balance for the first 6 days?

(d) What is the average daily balance for the first 10 days?

(e) If the daily balances in the checking account remained the same at $2400 the rest of the month, what is the average daily balance for the month?

(f) If the bank pays 5.25% per year in interest for the amount of money in the checking account, to be computed on the daily balance each day, what is the amount of interest that the bank pays for the money in the checking account on the month of June?

5. The charge card account of a person had the following amounts for the month of June:

From June 1 through June 8, the daily balance was $2000;
from June 9 through June 10, the daily balance was $2600;
from June 11 through June 15, the daily balance was $3000;
from June 16 through June 30, the daily balance was $3400.

(a) Find the average daily balance of the account for the first 6 days.

(b) Find the average daily balance of the account for the first 10 days.

(c) Find the average daily balance of the account for the first 20 days.

(d) Find the average daily balance of the account for the month.

(e) If the company that issued the charge card imposes the "finance charge" of 16% per year, to be computed on the daily balance, compute the finance charge imposed on the account for the month.

6. In high schools, colleges, and universities, each student carries what is called "Grade Point Average (GPA)" or "Grade Point Ratio (GPR)" to indicate his or her performance through the school, and in high schools, it is used to rank the students. It is computed as follows:

The grade of A is given 4 points;
the grade of B is given 3 points;
the grade of C is given 2 points;
the grade of D is given 1 point;
other grades like W (withdrawal), I (incomplete), F (not passing), etc., are given no point.

The assigned numbers 4, 3, 2, and 1, are called the "grade points". So, the grade point average is the weighted average of these points with the weights being the numbers of credits earned with the respective grades.

For example, if a student has 6 credits of A's, 8 credits of B's, 4 credits of C's, and 2 credits of D's, and no other grade, then

The GPA of the student =

From this number we estimate that overall the student is doing close to B work.

It should be stated that the computational procedure of the GPA varies from school to school, but for those with the grades of A, B, C, and D, the above computational procedure is standard.

(a) If another student has 10 credits of A's, 8 credits of B's, and 2 credits of C's, what is his grade point average?

(b) If a third student has 10 credits of A's, 10 credits of C's, and no other grade, what is his grade point average?

(c) A fourth student had GPA of 3.5 up to the end of her sophomore year with the total of 14 credits. In the junior year she buckled down and received 5 credits of A's and 1 credit of B. What is her GPA at the end of her junior year?

(d) If the student of (c) carries 6 credits during her senior year, what is the maximum (cumulative) GPA she can get?

(e) If a student has the GPA of 3.2 with 20 credits of courses taken and is carrying 15 credits of courses this semester, what is the minimum GPA for this semester in order to raise his GPA to 3.5?

(f) Another student has the GPA of 3.2 with 45 credits of courses taken and is carrying 15 credits of courses this semester. What should be the minimum GPA for this semester in order to raise his GPA to 3.5?

(g) Assuming that the student of (f) can do, on the average, 3.9 work for the rest of the courses, how many credits does he need to raise his GPA to 3.5? (His current GPA is 3.2 with 45 credits of courses.)

7. For a fund raiser, a certain club conducted a kalua pig sale. The club members prepared 600 pounds of kalua pig and sold 450 pounds at $7 per pound. The rest of the kalua pig was sold at a reduced price of $4 per pound, the next day.

(a) In effect, how much a pound did they sell?

(b) If they were to attain their goal, they had to sell at an average price of $6.50 per pound. Then, what should have been the price of the day-old kalua pig?

8.(a) If a student mixes 200 cc (cubic centimeters) of 10% hydrochloric acid solution with 600 cc of 40% hydrochloric acid solution, what is the concentration of the resulting solution?

(b) If a student has 150 cc of 10% hydrochloric acid solution and wants to raise its concentration to 20% by adding a suitable amount of 40% hydrochloric acid solution, how many cc's of the 40% solution does he have to add?

(c) If another student has 60 cc of 40% hydrochloric acid solution and lower its concentration to 30% by adding a suitable amount of water, then how many cc's of water does he have to add?

9.(a) A salt solution contained 7 grams of salt per liter of the solution. A student wanted to reduce the salt concentration of the solution, and so he added 5 liters of fresh water to 3 liters of the original solution. What is the salt concentration of the new solution?

(b) If the student of (a) wants to reduce the salt concentration of the solution, whose salt concentration is 7 grams per liter, to 3 grams per liter, how much water should he add to 2 liters of the original solution?

(c) An 80-liter tank is filled with the solution whose salt concentration is 7 grams per liter. If we want to reduce the salt concentration of the solution to 4 grams per liter by draining a suitable amount of the solution and replacing with water, how many liters of the solution should we drain and replace it with water?

10. A salt water aquarium contains 80 gallons of salt water whose concentration is 0.35 pound of salt per gallon of the salt water.

(a) If we take out 5 gallons of the salt water from the aquarium and put back the same amount of freshwater, what is the salt concentration of the salt water in the aquarium?

(b) If we want to reduce the concentration of the salt water in the aquarium to 0.29 pound of salt per gallons of the salt water (which is an approximate concentration of ocean water), how many gallons of the salt water do we have to remove from the aquarium and replace with the same amount of freshwater?