Chapter 5 Functions

One of the concepts that pervade all levels of mathematics is that of functions. Roughly speaking, a function is a definite relationship between two or more quantities. We have already encountered many functions although we did not use the word. For example, in the compound interest problems, the principal depends on the initial amount, the interest rate, and the number of years. We say that the principal is a function of the initial amount, the interest rate, and the number of years.

More precisely, we define a function as follows:

> A function is a rule of correspondence between elements (objects) of one set (collection) with elements of another set.

Often the rule of correspondence is defined by a mathematical expression like $x^2 - 3x + 1$. The rule of correspondence such an expression defines is between the value of x and the value of the expression. For this reason the mathematical expression itself is often spoken of as a function. We will be dealing mostly with the functions that are defined by mathematical expressions or equations.

5.1 Linear functions

We now examine the relationship between quantities x and y described by an equation of the type

$$y = mx + b$$

Since such an equation establishes the correspondence between x and y (that is, given a value of x, we can find the value of y), we say that such an equation defines y as a function of x.

We first examine the significance of the constants m and b in the equation . To get a concrete picture, let us look at an example, , and make a table of values for x and y, and plot the graph. (Make the table for yourself by copying the values of x and then computing the corresponding values of y). We obtain the table given on the right:

x	y
0	3
1	5
2	7
3	9
4	11
5	13

Table 1

We make the following observations:

(I) The 3 in is the value of y when . In fact, it is readily seen that if the relationship between x and y is given by the equation , the b in is the value of y when .

(II) As we increase the value of x by one unit, the value of y increases by 2 units (or more precisely, the value of y **changes** by units). In general, if the relationship between x and y is given by the equation , then as we increase the value of x by 1 unit, the value of y changes by m units (increases or decreases depending on the algebraic sign of m). That is, m, the coefficient of x, is the **rate of change** of y with respect to x.

These observations enable us to obtain the equation from the table that describes the linear relationship. For example, suppose that the relationship between x and y is given by the following table:

x	y
0	13
1	17
2	21
3	25
4	29

Table 2

We see that when , , and that the rate of change of y with respect to x is 4 (that is, as x increases by 1 unit, y changes by 4 units). Therefore, the equation that describes the relationship between x and y is .

Let us get back to the first example. When we plot the points given in Table 1 on a graph paper, we will notice that all the points seem to lie on a straight line. In fact, if we plot more points, even assigning fractional values to x, we will see that they all lie on the same straight line. The totality of the points whose coordinates satisfy the equation is called the **graph of the equation**. The graph of the equation is shown in Fig. 1.

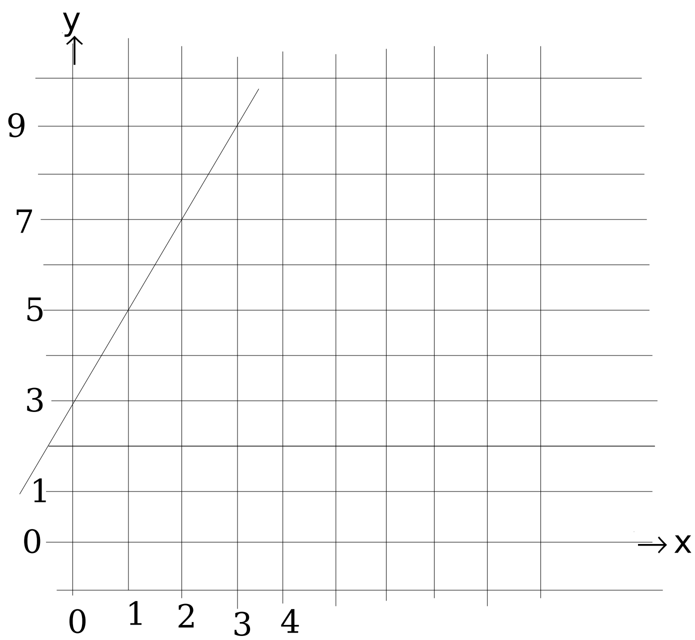


Fig. 1

In this case, the graph of the equation is a straight line. In fact, it turns out that the graph of an equation of the type is a straight line. It is for this reason that the equation of the type is called a **linear equation,** and we say that x and y are **linearly related** or that y is a **linear function** of x.

In general, by the **graph of a function f(x),** we mean the graph of the **equation** **.** That is, the graph of a function f(x) is the totality of the points (x,y) that satisfy the equation .

Exercises 5.1

1. Find the corresponding values of y to complete the table in each case and then plot the graph of the equation:

(a) $y = 3x - 14$

x	y
0	
1	
2	
3	
4	

(b) $y = -2x + 9$

x	y
0	
1	
2	
3	
4	

45 .

(c)

x	y
0	
1	
2	
4	
6	

(d)

x	y
0	
4	
8	
12	
16	

2. In each case find the equation that describes the relationship shown in the table:

(a)

x	y
0	23
1	28
2	33
3	38
4	43

(b)

t	d
0	15
1	11
2	7
3	3
4	-1

(c)

p	q
0.0	-5.0
1.0	-3.5
2.0	-2.0
3.0	-0.5
4.0	1.0

(d)

x	y
0	17
5	19
10	21
15	23
20	25

(e)

w	z
0.0	1.50
0.2	1.65
0.4	1.80
0.6	1.95
0.8	2.10

(f)

t	s
0	45
10	43
20	41
30	39
40	37

3. Find the equation that describes the relationship given the following statement:

(a) A car rental company charges its customers a fixed charge of \$45 per day plus the mileage charge of 9¢ per mile.

Let x = the number of miles a customer drives in one day, and
y = the charge for the day.

Fill in the table:

x	y
0	45
1	
2	
3	

(b) Suppose that the car rental company of (a) imposes 4% sales tax to the total charge computed in (a). Let z represent the total charge computed in (a) plus the 4% sales tax. Find the equation that describes z in terms of y first and then in terms of x.

(c) If a customer drives 120 miles in one day, what is the total amount, including the tax, that he has to pay?

(d) If another customer's total amount, including the tax, was \$60.84, how many miles did he drive?

4. The residents of a certain city are billed for the water usage according to the following schedule:

Meter charge --- \$18.50 per month.
90¢ per 1000 gallons of water used.

Let x = the number of gallons of water used in one month,
y = the charge for the month.

(a) Fill in the table:

47 .

x	y
0	
1000	
2000	
3000	

(b) Find the equation that expresses the relationship between x and y.

(c) If a household used 56,000 gallons of water in one month, what was charge for the month?

(d) If the charge for another household was $88.70, how many gallons of water did the household use in the month?

5. One of the popular stores in town has a gigantic annual sale in which the price of every item is reduced by 30%. On top of that, an employee of the store can get an additional 15% discount. A 4% State sales tax is added on to any purchase. That is, if an employee of the store buys and item, the store reduces the price of the item by 30%, then gives an additional 15% discount, and then adds the 4% State sales tax to compute the total amount that the employee pays for the item.

Let x = the regular price of an item,
y = the sale price of the item,
z = the employee price of the item,
u = the total amount that an employee pays for the item.

(a) Express y, z, and u as functions of x. (That is, express y as a function of x. Then, express z as a function of y first, and then express z as a function of x. Do the same for u.)

(b) If an employee of the store bought a dress whose regular price was $80, what was the total amount that the employee paid for the dress?

(c) If another employee of the store bought a pair of shoes and paid $77.35, (which included the 4% State sales tax), what would have been the regular price of the shoes?

5.2 The General Form

A more useful form of the equation describing a linear relation is given by $y = m(x - a) + c$. Again, let us see the significance of the constants a, c, and m by looking at an example, say . We see that it does not require any computation to find the value of y when , and that when the values of x are near 15, the values of y are near 8. So, we make the table of values starting at :

x	y
15	8
16	11
17	14
18	17
19	20

Table 3

We see that in the general case $y = m(x - a) + c$, the c is the value of y when $x = a$, and that the significance of m is the same as in the previous case, namely that m is the rate of change of y with respect to x. Again one of the reasons why we examine the significance of the constants a, c, and m in the equation $y = m(x - a) + c$ is that we want to be able to obtain the equation of the linear relation when the relationship is given by a table. For example, suppose that the relationship between two quantities x and y is given by Table 4.

x	y
40	17
45	20
50	23
55	26
60	29

Table 4

We see that as x increases by 5 units, y increases by 3 units, so that the per unit increase of y with respect to x is 3/5. This is the value of m. So, the equation that describes the relationship shown in the table is . It is always a good practice to check whether this equation does give the rest of the values in the table by evaluating the expression at one more value of x, say at . We see that

49 .

=

= 3 + 17

= 20

Thus it does give the next pair (45,20), and in fact we can visualize that the equation does give the rest of the values.

We note that there are various ways of writing the equation describing the relationship given in Table 4. We can use any of the pairs, (40,17), (45,20), (50,23), etc., as the "reference point" to write the equations:

and so on.

All of these equations can be reduced to .

Exercises 5.2

1. Find the corresponding values of y to complete the table in each case:

(a) $y = 2(x - 5) + 9$

x	y
5	
6	
7	
8	
9	

(b) $y = -3(x - 7) + 10$

x	y
7	
8	
9	
10	
11	

(c)

x	y
-4	
-3	
-2	
0	
2	

(d)

x	y
6	
11	
16	
21	
26	

2. In each case find the equation that describes the relationship shown in the table:

(a)

x	y
25	17
26	21
27	25
28	29
29	33

(b)

x	y
50	28
55	31
60	34
65	37
70	40

(c)

u	v
19	50
22	49
25	48
28	47
31	46

(d)

s	t
600	37
650	40
700	43
750	46
800	49

(e)

p	q
1970	11,000
1974	12,500
1978	14,000
1982	15,500
1986	17,000

(f)

t	w
3.5	1.0
3.5	2.3
3.9	3.6
4.1	4.9
4.3	6.2

3. A salesman at a certain company is paid \$800 per month plus the commission that is

5% of the amount over $10,000 of his sales for the month. (Assume that the amount of his sales is at least $10,000.)

Let x = the amount of his sales in one month, and
y = his pay for the month.

(a) Fill in the table:

x	y
10,000	
10,100	
10,200	
10,300	
10,400	

(b) Find the equation that describes the relationship between x and y.

(c) If a salesman of the company sold $28,600 worth of goods, what was his pay for the month?

(d) In the month of May, one employee of the company wants to earn $3,450. What should be his sales for the month of May?

4. In a certain city the real estate property is taxed at the rate of $6 per $1000 on the amount that is the assessed value of the property less $50,000 exemption.
Let V = the assessed value of a property, and
T = the tax on the property.

(a) Fill in the table:

V	T
50,000	
51,000	
52,000	
53,000	

(b) Find the equation that expresses the relationship between V and T.

(c) If the assessed value of a property is $275,000, what is the tax on the property?

(d) If the tax on a property is $1,827, what is the assessed value of the property?

5.3 Piece-wise Linear Functions

Often the relationship between two quantities is defined by two or more equations. Let us look at an example. The federal income tax for a married couple filing a joint return (as of 1991) was based on the following schedule:

If the taxable income is	the tax is
over $0 but not over $34,000	15% of the taxable income
Over $34,000 but not over $82,150	$5,100 plus 28% of the amount over $34,000
Over $82,150	$18,582 plus 31% of the amount over $82,150.

The relationship between the taxable income and the tax was defined by three equations. To express the relationship in equations, we let

x = the taxable income (in dollars), and

y = the tax determined by the above schedule.

Since the equations describing the relationship between x and y depend on the values of x, we have to specify the values of x as well. Then, the relationship between x and y described by the above schedule is given by the following equations:

$$y = 0.15x \qquad \text{if } 0 < x \le 34000$$
$$y = 5100 + 0.28(x - 34000) \qquad \text{if } 34000 < x \le 82150$$
$$y = 18582 + 0.31(x - 82150) \qquad \text{if } x > 82150$$

We note that if we compute the value of y for x = 34000 using the first equation, we obtain

$$y = 0.15(34000) = 5100$$

which is what we obtain when we set x equal to 34000 in the second equation. That is, the second equation actually holds for the lower end value 34000 as well. The same holds for the third equation.

When we draw the graph of the function described by the three equations, we obtain a graph like the one shown in Fig. 1.

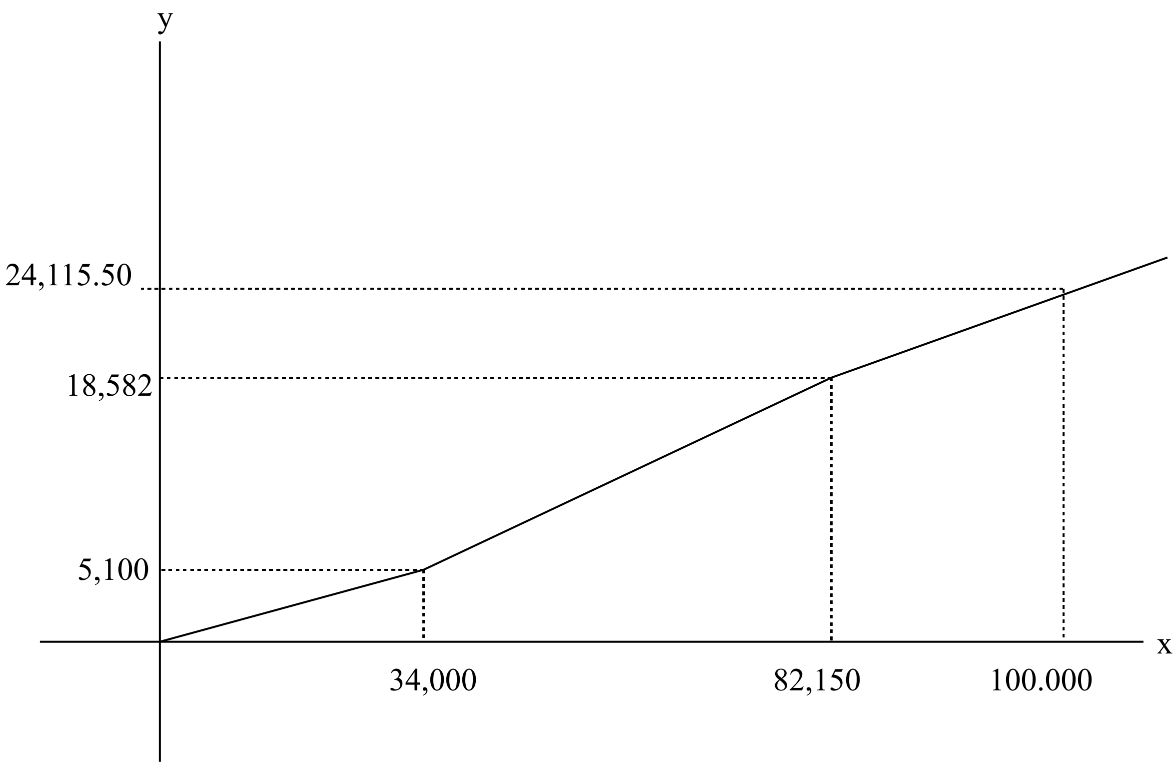


Fig 1

We have carefully indicated the points where the rate changes occurred. That is because we can recover the equations from such a graph. For example, to recover the second equation, we make the table

x	y
34,000	5,100
82,150	18,582

from which we obtain

=

=

We call a function of this type (that is, a function whose graph consists of line segments) a **piece-wise linear function.** Such functions are quite common in actuality. They are often used to approximate more complicated functions. In fact, in the days when we used to evaluate trigonometric functions, logarithmic functions, exponential functions, etc., by looking up their values in their tables, we were actually dealing with piece-wise linear functions that approximated the functions. For, whenever a particular value of such a function was not given in the table, we had to find the value by means of a method called **interpolation**. Let us look at how this was done.

Exercises 5.3

1. A salesman of a company earns a basic salary of $800 plus the commission that depends on the amount of sales he makes for the month and that is computed as follows:

   If his sales for a month is less than $10,000, he does not get a commission.
   If his sales of a month is over $10,000, his commission is

   5.5% of the first $20,000 over $10,000, plus
   8% of the next $20,000, plus
   11% of the amount over $50,000 (if his sales for the month is over $50,000.)

For example, if his sales for a month is $57,750, then

His pay = 800 + (0)(10,000) + 0.055(20,000) + 0.08(20,000) + 0.11(7,750) = 4,352.50

or $4,352.50.

Commission rate:	0%		5.5%		8%		11%
His sales:		10,000		30,000		50,000	

Let x = the amount of his sales for a month, and
y = his pay for the month..

(a) Fill out the tables:

x	y

x	y

x	y

55 .

10,000	800	30,000	50,000
10,100		30,100	50,100
10,200		30,200	50,200
10,300		30.300	50.300

30,000	50,000

(b) From the statement of the problem, get the first equation, (that is, when $0 \le x \le 10,000$).

(c) Using the first table, obtain the second equation. Using this equation, obtain the value of y when x = 30,000. Put the value of y for the first table and the second table.

(d) Fill in the second table and obtain the equation for the second table. Using this equation, obtain the value of y for x = 50,000. Enter the value in the second table and the third table.

(e) Fill in the third table and obtain the equation for the third table.

(f) Summarize the equations, stating the values of x for which each equation holds.

(g) If a salesman of the company had the sales of $57,750 in one month, using one of the equations, compute his pay for the month. If your value is not $4,352.50, you made a mistake, and so go back and check.

2. The electric company of a certain city imposes the following charges for the residential use of electricity:

Service charge ----- $6.00 per month

Up to 200 kwh at 11¢ per kwh (kilowatt hours)

Next 400 kwh at 9¢ per kwh

Next 600 kwh at 7¢ per kwh

Over 1200 kwh at 8¢ per kwh

(a) By constructing tables as in the case of Problem 1, find the equations that describe the total charge as a function of the amount of electricity used in a month. (You should have 4 equations.)

(b) If one household used 550 kwh of electricity in one month, what was the total charge for the month?

(c) If the total charge for the use of electricity was $73.10 for a certain month, how many kilowatt hours of electricity did the household use in the month?

3. The gas company of a certain city charges the residents for the use of the gas

according to the following schedule (per month):

First 3 therms at \$2.00 per therm
Next 7 therms at \$1.30 per therm
Next 10 therms at \$1.20 per therm
Next 10 therms at \$1.10 per therm
Over 30 therms at \$1.15 per therm
Minimum charge in a month is \$6.00.

(a) Find the equations that describe the total charge as a function of the amount of gas used in a month. (Note that the cost of the first 3 therms is \$6.00 --- the minimum charge. A household pays this much whether it uses 3 therms of gas or not.)

(b) If a household used 16 therms of gas in one month, what was the charge for the month?

(c) If the charge for another household was \$35.90, how many therms of gas did the household use in the month?

5.4 Computation of the interest rate

We now turn to the problem of finding the (true) interest rate for the installment purchase problem when the amount financed, the length of the loan, and the monthly payments are given. This is the most difficult problem associated with the formula because it involves the solution of a very high degree polynomial equation. But first we look at a much simpler problem. The simpler problem is this:

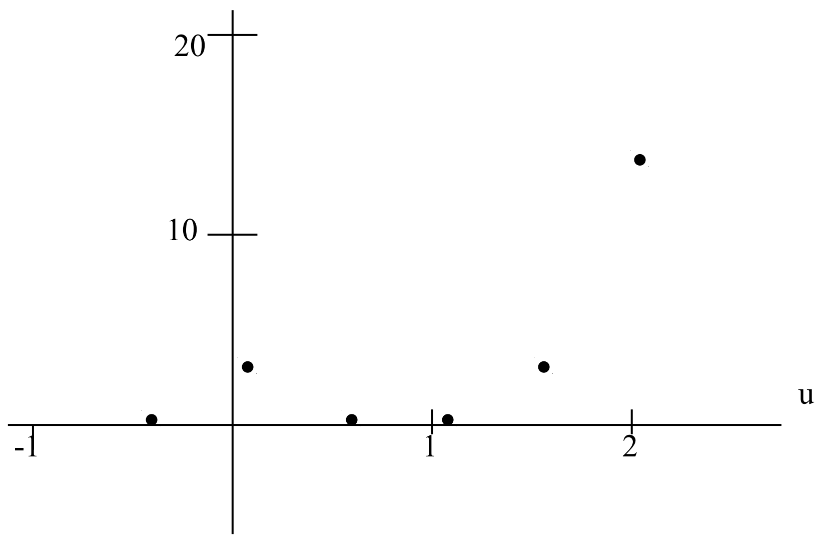
Find the solution, or rather approximate solution, of the equation,

$$6u^3 - 9u^2 + 1 = 0$$

that lies between 1 and 2.

When we construct a table of values of the function $f(u) = 6u^3 - 9u^2 + 1$ and sketch the graph, we see that the graph crosses the u-axis at a point between 1 and 1.5 or at about 1.4.

u	f(u)
-0.5	-2
0	1
0.5	-0.5
1	-2
1.5	1
2	13



We take 1.4 as the first approximation to the solution and use the following function to refine our approximation

$$F(u) = u - \frac{6u^3 - 9u^2 + 1}{18u^2 - 18u}$$

We let $u_1 = 1.4$

$u_2 = F(u_1) = F(1.4)$
$u_3 = F(u_2)$
$u_4 = F(u_3)$
$u_5 = F(u_4)$
and so on.

We continue this process until what we put in comes out unchanged or approximately unchanged. This value will be the solution of the equation $6u^3 - 9u^2 + 1 = 0$. (Actually it is the best approximation to the solution that our calculator can produce.)

Now we proceed with the computations:

$$u_2 = F(u_1) \;=\; F(1.4) \;=\; 1.4 - \frac{6(1.4)^3 - 9(1.4)^2 + 1}{18(1.4)^2 - 18(1.4)} \;=\; 1.417460317$$

$$u_3 = F(u_2) \;=\; u_2 - \frac{6(u_2)^3 - 9(u_2)^2 + 1}{18(u_2)^2 - 18(u_2)} \;=\; 1.41699363$$

$$u_4 = F(u_3) \;=\; u_3 - \frac{6(u_3)^3 - 9(u_3)^2 + 1}{18(u_3)^2 - 18(u_3)} \;=\; 1.41699329$$

$$u_5 = F(u_4) \;=\; u_4 - \frac{6(u_4)^3 - 9(u_4)^2 + 1}{18(u_4)^2 - 18(u_4)} \;=\; 1.41699329$$

Since u_4 and u_5 are the same, u_4 is the best approximation to the solution of the equation $6u^3 - 9u^2 + 1 = 0$ obtainable with our calculator. We can check by computing $f(u_4)$:

$$f(u_4) = 6(u_4)^3 - 9(u_4)^2 + 1 \;=\; 0$$

My calculator says $f(u_4)$ is equal to 0. But often $f(u_4)$ is equal to a number very close to 0.

Why does the method work? To answer the question we reformulate the problem:

> Finding the solution of the equation $6u^3 - 9u^2 + 1 = 0$ is the same as finding the point where the graph of the function $f(u) = 6u^3 - 9u^2 + 1$ crosses the u-axis.

To find this we first estimate this point and approximate the graph of the function near the point by a straight line, namely by the tangent line, and we take the next approximation to be the point where the tangent line crosses the u-axis. For example, in the above problem we took the first approximation to be 1.4. When we construct the tangent line at the point on the graph where u = 1.4, we find that the tangent line crosses the u-axis at the point $u = F(1.4)$. We take this value to be our second approximation u_2. We repeat this process. Graphical illustration of the first two steps is given in the figure below.

Now how do we get the function F(u)? We can explain the process to some extent but not fully because we cannot explain here how we get the slope of the tangent to the graph at $u = 1.4$ and in general at $u = u_k$. But let us see how far we can go.

Let m_1 be the "slope" of the tangent line at $u = u_1$. Then, the equation of the tangent line at the point of the graph where $u = u_1$ is

$$y = m_1(u - u_1) + f(u_1)$$

We want the value of u for which y = 0. So, we have to solve the equation

$$0 = m_1(u - u_1) + f(u_1)$$

for u. When we do that, we get

$$u = u_1 - \frac{f(u_1)}{m_1}$$

We take this value of u as the second approximation u_2. So,

$$u_2 = u_1 - \frac{f(u_1)}{m_1}$$

Now the question is how do we get the slope m_1? This is the topic that one can learn in the first two weeks of Calculus I, our Math 205, and it is not difficult. For our function $f(u)$, it turns out to be

$$m_1 = 18(u_1)^2 - 18(u_1)$$

So, we finally get

$$u_2 = u_1 - \frac{f(u_1)}{18(u_1)^2 - 18(u_1)}$$

To get the third approximation u_3, we go up to the graph of $f(u)$ and construct the tangent line and determine the point where the tangent line crosses the u-axis, and we obtain

$$u_3 = u_2 - \frac{f(u_2)}{18(u_2)^2 - 18(u_2)}$$

We let

$$F(u) = u - \frac{f(u)}{18u^2 - 18u}$$

We can describe the computational process by

$$u_2 = F(u_1)$$

$$u_3 = F(u_2)$$

$$u_4 = F(u_3)$$

$$u_5 = F(u_4)$$

and so on.

We hope that this explanation satisfies your curiosity as to how we obtain the "iterating function" $F(u)$.

Exercise:

There are two more values of u that make $f(u)$ equal to 0. One is near -0.3 and the other one is near 0.3. To find the best approximations our calculator can give, let $u_1 = -0.3$ for the first case and $u_1 = 0.3$ for the second case, and find the best approximations using the iterating function used in the above example.

We now get back to the problem of finding the (true) interest rate for the installment purchase problem when the amount financed, the length of the loan, and the monthly payments are given. The problem is this:

> When we finance P_0 dollars and make the monthly payment of M dollars for n months, what is the annual interest being charged?

Using the installment purchase formula

$$P_K = P_0\left(1+\frac{r}{12}\right)^k - \frac{M\left[\left(1+\frac{r}{12}\right)^k - 1\right]}{\frac{r}{12}}$$

we have to find r for which

$$0 = P_0\left(1+\frac{r}{12}\right)^n - \frac{M\left[\left(1+\frac{r}{12}\right)^n - 1\right]}{\frac{r}{12}}$$

To simplify the writing, we let $u = 1 + \frac{r}{12}$, so that $\frac{r}{12} = u - 1$, and so the equation becomes

$$0 = P_0(u)^n - \frac{M[u^n - 1]}{u - 1}$$

Simplifying this equation, we obtain

$$P_0(u)^{n+1} - (P_0 + M)u^n + M = 0$$

So, our $f(u)$ is

$$f(u) = P_0(u)^{n+1} - (P_0 + M)u^n + M$$

and our iterating function $F(u)$ is

$$F(u) = u - \frac{P_0(u)^{n+1} - (P_0 + M)u^n + M}{(n+1)P_0(u)^n - n(P_0 + M)u^{n-1}}$$

Let us see how we can use these equations.

Example 1: A car salesman says that you can have a $15,000 brand new car with the monthly payment of $300 for 5 years. What is the annual interest rate that the company is charging?

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Solution: We have $P_0 = 15000$, $M = 300$, and $n = 5(12) = 60$, so that the equation to solve is

$$15000u^{61} - 15300u^{60} + 300 = 0$$

and the iterating functions is

$$F(u) = u - \frac{15000u^{61} - 15300u^{60} + 300}{61(15000)u^{60} - 60(15300)u^{59}}$$

Now how do we choose the first approximation? We just have to guess. We figure that the annual interest rate is no more than 12%. The corresponding value of r is $u = 1 + \frac{0.12}{12} = 1.01$. So, we can take 1.01 as our first approximation. Therefore, we have

$$u_1 = 1.01$$

$$u_2 = F(u_1) = u_1 - \frac{15000(u_1)^{61} - 15300(u_1)^{60} + 300}{61(15000)(u_1)^{60} - 60(15300)(u_1)^{59}}$$

$$= 1.007514434$$

$$u_3 = F(u_2) = 1.0064325$$

$$u_4 = F(u_3) = 1.0061949$$

$$u_5 = F(u_4) = 1.0061834$$

$$u_6 = F(u_5) = 1.0061834$$

$$u_7 = F(u_6) = 1.0061834$$

Recall that $u = 1 + \frac{r}{12}$, and so

$$1 + \frac{r}{12} = 1.0061834$$

$$r = 12(0.006183413) = 0.0742009$$

Therefore, the annual interest of the financing is about 7.42%.

Let us check our answer. We substitute our data into the formula for the computation of the monthly payment with our computed interest rate:

$$M = \frac{15000\left(1+\frac{0.0742}{12}\right)^{60}\left(\frac{0.0742}{12}\right)}{\left(1+\frac{0.0742}{12}\right)^{60} - 1} = 299.999318$$

which is very close to the given monthly payment, and so our result is correct.

Example 2: The Low-Cost Loan Company offers fast loans up to \$9,000 at only 6% per year, simple interest. A man walks into the office and asks for a loan of \$9,000 for two years. The loan officer deducts the interest 9,000(2)(0.06) = 1080 dollars and gives him \$7920, telling him that the monthly payment is $\frac{9000}{24} = 375$ dollars over the next two years. Compute the true annual interest rate the company is charging.

Solution: As far as the man is concerned, he is borrowing \$7920 and makes the monthly payment of \$375 for 24 months. So, the equation to solve is

$$7920u^{25} - (7920+375)u^{24} + 375 = 0$$

or

$$7920u^{25} - 8295u^{24} + 375 = 0$$

The iterating function is

$$F(u) = u - \frac{7920u^{25} - 8295u^{24} + 375}{25(7920)u^{24} - 24(8295)u^{23}}$$

We can take the first approximation to be $1+\frac{0.24}{12} = 1.02$, which corresponds to the true annual interest rate of 24%. (You may choose another first approximation and see the difference.). Therefore, with $u_1 = 1.02$, we have

$$u_2 = F(u_1) = u_1 - \frac{7920(u_1)^{25} - 8295(u_1)^{24} + 375}{25(7920)(u_1)^{24} - 24(8295)(u_1)^{23}}$$

$$= 1.014140115$$

$$u_3 = F(u_2) = 1.0113630$$

$$u_4 = F(u_3) = 1.0105621$$

$$u_5 = F(u_4) = 1.0104904$$

$$u_6 = F(u_5) = 1.0104899$$

$$u_7 = F(u_6) = 1.0104899$$

Therefore,

$$1 + \frac{r}{12} = 1.0104899$$

$$r = 12(0.0104899) = 0.125879$$

Therefore, the true annual interest rate is about 12.588%. Let us check our answer by computing the monthly payment with this interest rate:

$$M = \frac{7920\left(1+\frac{0.12588}{12}\right)^{24}\left(\frac{0.12588}{12}\right)}{\left(1+\frac{0.12588}{12}\right)^{24} - 1} = 375.00038$$

which is very close to the required monthly payment, and so our answer is correct.

Exercises 5.4

1. (a) If a man obtains a loan of \$6000 at 5% per year, simple interest, for 4 years, to be repaid by monthly installments, compute the true annual interest rate of the loan. (Here, the loan officer deducts the interest, which is 6000(0.05)(4) = 1200 dollars, from \$6000, so that the man gets only \$4800, but he makes the monthly payment of $\frac{6000}{48} = 125$ dollars.)

   (b) Check your answer of (a) by computing the monthly payment with that interest rate.

2. A newspaper advertisement says that you can have an \$18,000 brand new car for the monthly payment of \$400 for 5 years. What is the annual interest rate of the financing?

3. Easy Loan Company offers fast loans at only 10% per year, simple interest. A man

walks into the office and asks for a loan of $6,000 for two years. The loan officer deducts the interest dollars and gives him $4800, telling him that the monthly payment is dollars over the next two years. What is the true annual interest rate of the loan? (Remember that from the customer's point of view, he is getting $4800 and makes the monthly payment of $250 for two years.)

4. The Better-Home Loan Company charges 2 points for home mortgage loans at the interest rate of 6.75% per annum. (That is, the customer must pay the company 2% of the amount of the loan to secure the loan.) If a man obtains a 30-year loan of $400,000 from the company to buy a house, what will be the effective true annual interest rate? (Since he has to pay 400,000(0.02) = 8,000 dollars, the amount he gets is 400,000 – 8,000 = 392,000. The effective true annual interest rate should be computed on this amount.) The computational procedure is as follows:

(a) Compute the monthly payment on the 30-year loan of $400,000 at 6.75% per annum.

(b) Compute the true annual interest rate if the man takes the 30-year loan of $392,000 and makes the monthly payment of the amount computed in (a).