Chapter 3 Linear Equations

3.1 Linear equations

We encounter many situations in which we have to solve linear equations (that is, equations in which the exponent of the unknown is 1). The skill to solve such equations is indispensable. So, we now review the method of solving such equations.

Consider the problem: What is the number x which when multiplied by 8 gives 125?

We know at once that the number x must be greater than 10 since 10 times 8 is 80 but less than 20 since 20 times 8 is 160. We can narrow down our estimates and obtain a reasonable answer in a few minutes. However, we want a more efficient method of finding the answer. To do so, we formulate the problem into an equation:

$$x(8) = 125 \quad \text{or}$$

Since 8 times x is to be 125, we reason that x must be the number that is obtained by dividing 125 by 8. That is,

.

So, the number is or . The number or is called **the solution of the equation** .

To solve more complicated problems, we make use of the following **properties of equality**:

(i) If equals are added to equals, the results are equal.

(ii) If equals are multiplied by equals, the results are equal.

Since subtraction of a number is defined to be the addition of its negative, the statement, "If equals are subtracted from equals, the results are equal." is included in Statement (i). Similarly, since division by a number is defined to be the multiplication by its reciprocal, the property of equality about division is included in Statement (ii).

We can formally solve the simple equation given above as follows:

Of course, we do not write in such great detail, but it is important to know the principles being used.

Example 1: Find the solution of the equation .

Solution: We will write out the solution in great detail. Once you get used to, you can omit many of the steps.

The idea is to bring the equation to the form ax = b by making use of the properties of equality. To get rid of from the left-hand side, we add 37 to both sides of the equation:

or

To get rid of from the right-hand side, we add to both sides of this equation:

or

and so

Example 2: Find the solution of the equation $5(3t+2) = 2(5t+7)+8$.

Solution: Here the first problem is to simplify both sides of the equation:

$$15t+10 = 10t+14+8$$

$$15t+10 = 10t+22$$

$$-10t+15t = 22-10$$

$$5t = 12$$

$$t = \frac{12}{5} = 2.4$$

Therefore, the solution of the equation is 2.4.

Example 3: Find the solution of the equation $8u - \frac{4(u-1)}{3} = 25$.

Solution: Here again the first step is to simplify the left-hand side of the equation. Combine the two terms into one fraction:

$$\frac{8u(3) - 4(u-1)}{3} = 25$$

$$(3)\frac{8u(3) - 4(u-1)}{3} = 25(3)$$

$$24u - 4u + 4 = 75$$

$$20u + 4 = 75$$

$$20u = 75 - 4$$

$$20u = 71$$

$$u = \frac{71}{20} = 3.55$$

The solution is 3.55.

Since the procedure involved many steps, we should check the solution by computing the left-hand side of the original equation for u = 3.55.

Exercises 3.1

1. Find the solution of each of the following equations: (You should check your solution at least for some of the problems.)

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k) $\frac{80}{x+2} = 5$

(l) $4u - \frac{2(u-1)}{3} = 15$

3.2 Literal equations

We often have to solve an equation for one of the variables in terms of the other variables. What we are doing in such cases is essentially postponing the substitution of the values of the variables to the last. Consider the following problem for example.

Example 1: A man wants to buy a new car priced at $18,000, financing it at 5.75% per annum and paying back the loan in monthly installments for 5 years. What is the monthly payments?

We could have substituted the values into the installment purchase formula

$$P_k = P_0\left(1+\frac{r}{12}\right)^k - \frac{M\left[\left(1+\frac{r}{12}\right)^k - 1\right]}{\frac{r}{12}}$$

and obtained the equation

$$0 = 18{,}000\left(1+\frac{0.0575}{12}\right)^{60} - \frac{M\left[\left(1+\frac{0.0575}{12}\right)^{60} - 1\right]}{\frac{0.0575}{12}}$$

We can solve this equation as follows:

$$0 = 23{,}979.16069 - M(69.32360218)$$

$$M(69.32360218) = 23{,}979.16069$$

$$M = \frac{23{,}979.16069}{69.32360218} = 345.901827$$

So, the monthly payment is $345.90.

Or we can substitute the values in the monthly payment formula

$$M = \frac{P_0\left(1+\frac{r}{12}\right)^n \times \frac{r}{12}}{\left(1+\frac{r}{12}\right)^n - 1}$$

and get $M = \dfrac{18{,}000\left(1+\frac{0.0575}{12}\right)^{60} \times \frac{0.0575}{12}}{\left(1+\frac{0.0575}{12}\right)^{60} - 1} = 345901827$

which is exactly the same as the value obtained above.

Let us review how we obtained the monthly payment formula from the installment purchase formula.

We let

$$A = P_0\left(1+\frac{r}{12}\right)^n$$

$$B = \left[\left(1+\frac{r}{12}\right)^n - 1\right]$$

$$C = \frac{r}{12}$$

Then, we have

$$0 = A - \frac{MB}{C}$$

We have to solve this equation for M. Adding $\frac{MB}{C}$ to both sides of the equation, we get

$$\frac{MB}{C} = A$$

Multiplying both sides of the equation by C and then dividing both sides of the equation by B, we obtain

$$M = \frac{AC}{B}$$

Substituting back, we get the monthly payment formula:

$$M = \frac{P_0\left(1 + \frac{r}{12}\right)^n \cdot \frac{r}{12}}{\left(1 + \frac{r}{12}\right)^n - 1}$$

Exercises 3.2

Solve each of the equations for the indicated variable:

(a) $A = Prt$ for r.

(b) $A = Prt$ for P.

(c) $A = P + Prt$ for P.

(d) $A = P + Prt$ for r.

(e) $x - rx = A$ for x.

(f) $Pu - \frac{M(u-1)}{B} = 0$ for M.

(g) $Pu - \frac{M(u-1)}{B} = 0$ for u.

(h) $Pu - \frac{M(u-1)}{B} = C$ for M.

(i) $Pu - \frac{M(u-1)}{B} = C$ for u.

(j) $A = Bu + \frac{x(u-1)}{C}$ for x.

(k) $A = Bu + \frac{x(u-1)}{C}$ for u.