

Foundations Course Articulation Renewal Proposal
Mathematics 140: *Pre-Calculus: Trigonometry & Analytic Geometry*
As an Symbolic Reasoning (FS) Course

Leeward Community College
Spring 2007

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I. Course Description.

- Title & Catalog Description

PRE-CALCULUS: TRIGONOMETRY & ANALYTIC GEOMETRY (3 credits): A study of the properties and graphs of trigonometric, circular, and inverse functions; solution of triangles; identities; solution of trigonometric equation; conic sections; polar coordinates; parametric equations

- Student Learning Outcomes

Upon successful completion of Math 140, a student should:

1. Understand and be able to calculate and apply the trigonometric ratios of acute angles.
2. Understand and be able to evaluate the trigonometric functions of variables expressed in general angle and radian units.
3. Be able to analyze and graph trigonometric functions.
4. Be able to solve application problems involving trigonometric concepts, including right triangles, arc length, area, and angular speed.
5. Understand the derivation of and be able to apply the essential trigonometric identities.
6. Be able to use the essential trigonometric identities to simplify expressions and prove further identities.
7. Understand and be able to analyze, graph, and use the inverse trigonometric functions.
8. Understand the derivation of, and be able to apply, the Law of Sines and Law of Cosines.
9. Understand and be able to use further trigonometric topics, including vectors in the plane, parametric equations, and polar coordinates.
10. Understand the properties of parabolas, ellipses, and hyperbolas.
11. Be able to solve various application problems related to trigonometry and the conic sections.

II. Changes.

No significant changes have been made in Math 140 since the original request for foundations designation was approved.

III. Assessing the Course: Below are samples of course materials that illustrate how the course meets the Foundations Hallmarks.

Foundations Hallmarks & Application Questions SYMBOLIC REASONING (FS)

1. **Students will be exposed to the beauty, power, clarity and precision of formal system. *How will the course meet this hallmark?***

Math 140 is a standard one-semester course in trigonometry and analytic geometry. Trigonometric functions of acute angles are first introduced to students. They are defined as ratios of the lengths of sides in right triangles. This gives instructors the opportunity to demonstrate how some abstractly derived trigonometric properties, relationships, and formulas can be visualized in application.

For example, the $30^\circ - 60^\circ$ right triangle theorem is usually covered right after the definitions of trigonometric functions of an acute angle are introduced. By using this theorem, the definitions, and also the famous Pythagorean theorem, which the students learned from earlier classes, the students will be able to find the exact values for all six trigonometric functions evaluated at 30° and at 60° . The knowledge of these values are fundamental to the entire course. Also at this point, students will learn the difference between an exact answer and an approximate answer. For instance, the exact value of $\sin 60^\circ$ is $\frac{\sqrt{3}}{2}$, while the approximated answer to ten decimal places provided by the calculators is 0.8660254038.

In the section of right-triangle applications, students will see how the knowledge of trigonometric functions can help to solve real world problems. The following are two sample problems.

- i. From a highway overpass, 14.3 m above the road, the angle of depression of an oncoming car is measured at 18.3° . How far is the car from a point on the highway directly below the observer?
- ii. From a point level with and 1000 ft away from the base of the Washington Monument, the angle of elevation to the top of the monument is 29.05° . Determine the height of the monument to the nearest half foot.

Radian measure and the general trigonometric functions are then introduced as an amazing generalization that yields students' first exposure to continuous periodic functions. Applications, such as finding the area of a sector, finding the angular speed and linear speed of a wheel rotating at a constant rate, describing simple harmonic motion, etc., are introduced to illustrate the utility of this new class of functions and their properties. These applications demonstrate to our students that the knowledge from this

course will help them to understand and solve physics problems. The following are some sample material extracted from class exercises and homework problems.

- i. A belt drives two wheels of radii $r = 10$ cm and $R = 18$ cm. It's given that the angular speed of the larger wheel is 360 rpm.
 - a. What is the angular speed of a point on the circumference of the larger wheel?
 - b. What is the linear speed of a point on the circumference of the larger wheel?
 - c. What is the angular speed of the smaller wheel in radians per minute?
 - a. What is the angular speed of the smaller wheel in rpm
- ii. Assume that the simple harmonic motion is described by the equation $s = 4 \cos\left(\frac{2t}{3}\right)$, where s is in feet, t is in seconds, and the equilibrium position of the mass is $s = 0$.
 - a. Specify the amplitude, period, and frequency for this simple harmonic motion, and sketch the graph of the function $s = 4 \cos\left(\frac{2t}{3}\right)$ over the interval $0 \leq t \leq 6\pi$.
 - b. When during the interval of time $0 \leq t \leq 6\pi$ is the mass moving upward?
 - c. When during the interval of time $0 \leq t \leq 6\pi$ is the mass moving downward?
 - d. For this harmonic motion, it can be shown (using calculus) that the velocity v of the mass is given by $v = -\frac{8}{3} \sin\left(\frac{2t}{3}\right)$, where t is in seconds and v is in ft / sec.
Graph this velocity function over the interval $0 \leq t \leq 6\pi$.
 - e. Use your graph of the velocity function from part (d) to find the times during this interval when the velocity is zero. At these times, where is the mass?
 - f. Use your graph of the velocity function to find the times when the velocity is maximal. Where is the mass at these times?
 - g. Use your graph of the velocity function to find the times when the velocity is minimal. Where is the mass at these times?

Students in Math 135 study the properties and graphs of several functions, including polynomial, rational, exponential, and logarithmic functions. In this course, students will extend their knowledge of functions to trigonometric functions. For example, students will first study the basic sine curve and cosine curve. They will then apply the translation and reflection properties of graphs of functions together with the definition of periodic functions to graph functions of the types $A \sin(Bx - C)$ and $A \cos(Bx - C)$ over several periods. Later, the same properties will be used to graph the rest of trigonometric functions. The properties of conic sections are also studied in the context of functions and relations.

The graphs of conic section were briefly covered in Math 135. In this course, the standard-form equations of parabola, ellipse and hyperbola are derived from their definitions as sets of points satisfying distance relationships. Their abstract and analytic properties are studied. Fascinating applications, such as the reflective properties of parabolas, are introduced to illustrate the practical aspects of this analytic approach.

- i. Graph $y = 3\sec\left(3x - \frac{\pi}{4}\right)$ over two periods. One of the periods should include $x = 0$.

Label the axes and the vertical asymptotes clearly. Determine the period, x - and y -intercepts. Indicate the intercepts and the local extreme points on the graph.

- ii. One of the largest radio telescopes in the world is the Effelsberg radio telescope, located 25 miles southwest of Bonn, Germany. Determine the diameter d of its parabolic reflector, given that the focal ration is 0.3 and the reflector maybe described by the parabola $y^2 = 120x$. (Assume that the units for x and y are meters.)

2. **Instructors will help students understand the concept of proof as a chain of inferences. *How will instructors help students understand this concept?***

The proofs of important theorems and properties are studied or discussed in class. The essential trigonometric identities and conic section standard-form equations are derived in class, as well. The importance of proving identities and the techniques involved is demonstrated and emphasized. Students are responsible for their own proofs of identities and derivations of formulas throughout the course.

For example, in the section of the addition formulas, the instructor will carefully prove the identity, $\cos(x + y) = \cos x \cos y - \sin x \sin y$, for the students first, and then will show the students how this identity can be used to prove the following two identities:

$\cos(x - y) = \cos x \cos y + \sin x \sin y$ and $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$. Then the instructor will let

students try to prove the addition formulas for sine function and tangent function using the three identities that are just proved with minimal help from the instructor. Class discussion among the students is encouraged at this point. Students may even think out loud making suggestions on what to do. Usually with very little hint from the instructor, the students are able to finish all the proofs by themselves. In the following section, the double-angle formulas and the half-angle formulas are introduced, and students are able to derive the formulas on their own using the addition formulas that they have just learned from the previous section with some direction from the instructor. Samples from students' in class practices and homework assignments, which involve proving identities are given below.

i.
$$\frac{1 - \cot^2 A + \cos^2 A \cot^2 A}{\sec^2 A - 1} = \cos^2 A$$

ii.
$$\frac{\csc A \cot A - \sec A \tan A}{\cos A - \sin A} = \frac{1 + \csc A \sec A}{\sin A \cos A}$$

iii. $\cos 3x = 4 \cos^3 x - 3 \cos x$

3. **Instructors will teach students how to apply formal rules or algorithms. *How will instructors meet this hallmark?***

Evaluating trigonometric functions of real numbers involves the following steps:

- i. Determine the quadrant in which the terminal side of the angle lies;
- ii. Determine whether the particular trigonometric function takes positive or negative values in that quadrant;
- iii. Determine the reference number of the angle;
- iv. Evaluate the function based on the information obtained from steps i to iii.

Instructor will show students why the above steps are necessary for evaluating trigonometric functions of real numbers by using the unit circle definitions of trigonometric functions. The instructor will then help students to learn the concepts through exercises such as the ones shown below.

- i. Evaluate $\sec \frac{17\pi}{3}$
- ii. Evaluate $\tan 510^\circ$

Also, a variety of problems involving triangles and other more general geometric figures are solved using formal rules and algorithms. Students must learn fundamental definitions of trigonometry and analytic geometry, and the formal properties that are derived from these definitions, then apply these definitions and properties to abstract as well as concrete problems. For example, finding the area of a sector, and then extend the concept to finding the area of an equilateral curved triangle. Students later will discover how these geometric figures may be used in gothic window design through homework assignments like the one below.

- i. Figure F shows a gothic window design from the cathedral at Reims, France. (The cathedral was constructed during the years 1211-1311.) Figure G indicates how the design is formed. Triangle ABC is equilateral and arch ABC is the corresponding equilateral arch. Equilateral triangle GID , congruent to triangle ABC , is constructed such that D is the midpoint of \overline{AB} , and \overline{GI} is parallel to \overline{AB} . The points E and F are the midpoints of segments \overline{DG} and \overline{DI} , respectively. The two small arches at the bottom of the figure are equilateral arches corresponding to the equilateral triangles GHE and HIF . For the circle in the figure, the center is D and the radius is AD ($= ED = FD$).
 - a. Let s denote the length of a side in each of the two equilateral triangles ABC and GID . Find the area and the perimeter (in terms of s) of the curved figure $AJBCA$.
 - b. Express (in terms of s) the area and the perimeter of equilateral arch GHE .
 - c. Show that the area of the curved figure EHF is $s^2(2\sqrt{3} - \pi)/8$.

- d. Express (in terms of s) the perimeter and the area of the curved figure FBI .

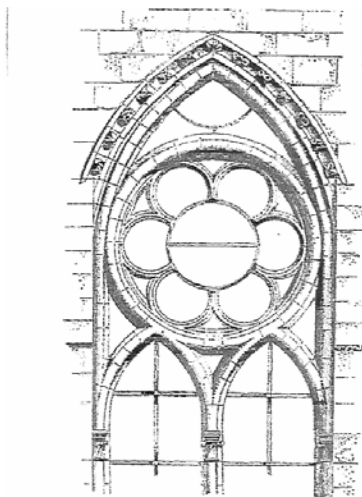


Figure F
A window design in the cathedral at Reims, France. From Hans H. Hofstätter, *Living Architecture: Gothic* (Office du Livre, S. A., 1970)

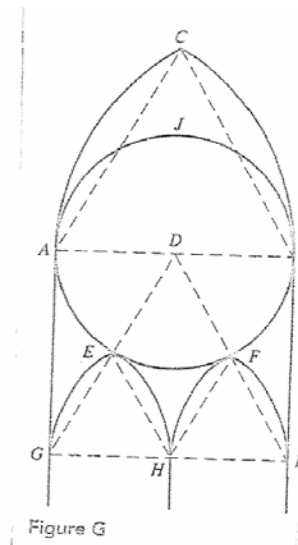


Figure G

Teaching students how to graph trigonometric functions is another example. The process of graphing these functions requires students to follow an algorithmic process of analyzing the domain, range, period, horizontal and/or vertical translation, and intercepts. Worksheets are designed to help students learn the concepts. Students are also required to select and apply appropriate trigonometric techniques to concrete problems.

The following are some more sample problems that require students to apply formal rules or algorithms.

- i. Graph the function $y = \frac{3}{2} \tan\left(\pi x + \frac{\pi}{4}\right)$ over two periods. Label the axes and the vertical asymptotes clearly. Determine the period of the function. Find the x - and y -intercepts, and indicate the intercepts on the graph.

ii. Simplify
$$\frac{\cos \frac{5\pi}{12} + \sin \frac{5\pi}{12}}{\cos \frac{\pi}{12} - \sin \frac{\pi}{12}}$$

- iii. Graph the parabola, $y^2 - 2y + 8x + 17 = 0$. Specify the vertex, the focus, the directrix, the focal width, the axis of symmetry, and the x - and y -intercepts.

4. **Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence. What symbolic techniques will be required and in what contexts? *How will presentations and evaluations be incorporated into the course?***

In the course of using trigonometric or analytic geometry to solve concrete problems, students must form a mathematical model of the situation, manipulate the model by applying appropriate techniques, interpret the results of the manipulations, and check the correctness of their results.

For instance, all the word problems are examples of this type. Every time when we try to solve a word problem, instructors will demonstrate the proper execution of each of the steps mentioned above in class: translate the English description of the problem into a simple drawing of the situation, indicate the given information on the graph, name the unknowns, identify what definitions, formulas, and / or theorems are needed in order to solve for the unknowns, apply the necessary algebraic computation, and then check the validity of the answer. Students are expected to follow a similarly comprehensive process in their homework and test/quiz problems. Below are some sample problems.

- i. Solve $\triangle ABC$ if $a = 10$, $b = 13$, and $A = 25^\circ$. Round your answer to one decimal place.
 - ii. The angle of elevation of the top of a water tower from point A on the ground is 19.9° . From point B , 50.0 feet closer to the tower, the angle of elevation is 21.8° . What is the height of the tower?
 - iii. Suppose that an object is attached to a coiled spring. It is pulled down a distance of 5 in. from its equilibrium position, and then released. The time for one complete oscillation is 4 sec.
 - a. Give an equation that models the position of the object at time t .
 - b. Determine the position at $t = 1.5$ sec.
 - c. Find the frequency.
5. **The course will not focus solely on computational skills. *What reasoning skills will be taught in the course?***

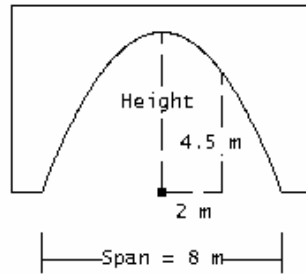
The major goals of this course are to expand the study of functions to the trigonometric functions, to analyze and manipulate the properties possessed by this new class of functions, and to abstractly analyze some geometric figures (the conic sections).

There is a wide class of problems where computational skills play only a part of the solution. For example, students must logically derive formulas or decide which method or algorithm is amenable to which problem. While solving real-world application problems, students must utilize their critical thinking ability to analyze the problems and to decide which theorems, formulas, etc., are applicable before they can even do any

computations. The following are some good examples of this type of problems. Without understanding the concepts, students may not know what to compute, what to do with the computational results, or even do not know where to start solving the problem. In conclusion, computations only play a secondary role in learning.

i.
$$\sin\left(\frac{\tan^{-1}\left(-\frac{5}{12}\right)}{2}\right)$$

- ii. Find the standard equation of the ellipse with foci $(-1, 4)$ and $(-1, 0)$, and the endpoints of the minor axis $(-6, 2)$ and $(4, 2)$. Identify the center and vertices. Graph the ellipse.
- iii. The span of the parabolic arch, see Figure below, is 8 m. At a distance of 2 m from the center, the vertical clearance is 4.5 m. Find the height of the arch.

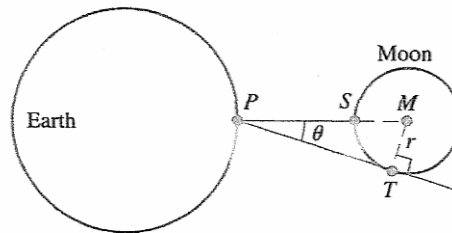


6. **Instructors will build a bridge from theory to practice and show students how to traverse this bridge. How will instructors help students make connections between theory and practice?**

As previously stated, the right triangles approach is used to introduce trigonometric ratios and functions. This allows instructors to show the motivation for abstract properties of trigonometric functions. Once these properties are established, instructors introduce and explain numerous real-world examples of application problems and questions that give rise to trigonometric models. After applying the properties to the model and drawing a conclusion from it, instructors show how the solution fits into the context of the original application problem or question.

Instructors introduce other topics, including the law of sines and the law of cosines, parametric equations, polar system, the analytic study of conic sections, in the abstract first, then show how the abstract properties used or derived give rise to interesting, useful applications, such as the reflective property of a parabola, and examples in navigation. As with the applications motivated by triangles, students are then challenged to identify and solve application problems on their own after their exposure to these examples. Below are some sample problems.

- i. Refer to figure below. Let r denote the radius of the moon.
 - a. Show that $r = \left(\frac{\sin \theta}{1 - \sin \theta} \right) PS$.
 - b. Use a calculator and the following data to estimate the radius r of the moon: $PS = 238,857$ miles and $\theta = 0.257^\circ$. Round your answer to the nearest 10 miles.



- ii. A small fire is sighted from ranger stations A and B. The bearing of the fire from station A is $N35^\circ E$, and the bearing of the fire from station B is $N49^\circ W$. Station A is 1.3 miles due west of station B. How far is the fire from each ranger station? At fire station C, which is 1.5 miles from A, there is a helicopter that can be used to drop water on the fire. If the bearing of C from A is $S42^\circ E$, find the distance from C to the fire, and find the bearing of the fire from C.
- iii. Describe the curve represented by the parametric equations $x = 2 \cos 2t$, $y = 2 \sin 2t$, $0 \leq t \leq 2\pi$.

Sample Syllabus

LEEWARD COMMUNITY COLLEGE
Mathematics and Natural Sciences Division
Course Syllabus
Math 140 – Pre-Calculus (3.0 credits)

Instructor: **Office Hours:** **Office Location:**
Phone: **E-mail:**

Catalog Course Description: A study of the properties and graphs of trigonometric, circular, and inverse functions; solution of triangles; identities; solution of trigonometric equations; conic sections; polar coordinates; parametric equations.

Co-requisites: None
Prerequisites: C or better in Math 135 or equivalent
Recommended Preparations: None

Textbook and other Resources:

- David Cohen, A Custom Edition of Precalculus, Prepared Exclusively for the Leeward Community College Mathematics Discipline.

Student Learning Outcomes:

Upon successful completion of Math 140, a student should:

- understand and be able to calculate and apply the trigonometric ratios of acute angles;
- understand and be able to evaluate the trigonometric functions of variables expressed in general angle and radian units;
- be able to analyze and graph trigonometric functions;
- be able to solve application problems involving trigonometric concepts, including right triangles, arc length, area, and angular speed;
- understand the derivation of, and be able to apply, the essential trigonometric identities;
- be able to use the essential trigonometric identities to simplify expressions and prove further identities;
- understand and be able to analyze, graph, and use the inverse trigonometric functions;
- understand the derivation of, and be able to apply, the Law of Sines and Law of Cosines;
- understand and be able to use further trigonometric topics, including vectors in the plane, parametric equations, and polar coordinates;
- understand the properties of parabolas, ellipses, and hyperbolas;
- be able to solve various application problems related to trigonometry and the conic sections.

Withdrawal Deadlines:

Other Important Dates:

Attendance: Students are expected to attend each class session on time. They are also expected to stay for the entire duration of the class. Anticipated as well as unexpected absences should be discussed with the instructor. The acceptance of an excuse for absence is at the discretion of the instructor. Students are expected to have their textbooks and note-taking equipment (writing instrument and paper) during each class session. Beyond being physically present in the room students are expected to be alert and engaged in the lecture or discussion. Students will be responsible for all material, discussion, and assignments covered during any missed class session(s). Students who miss class (for whatever reason) should arrange to obtain a set of class notes from a classmate, read through the covered sections and attempt the assigned problems. Students who miss a class will NOT be provided with a repeat of the missed lecture.

Grading Policy:

Course evaluation will be done through quizzes and examinations:

- In class quizzes are worth 5 points each. The best 10 scores will count toward the course grade.
- Take home quizzes are worth 10 points each. The best 10 scores will count toward the course grade.
- Midterms are worth 50 points each. There are 3 midterms. Exam dates will be announced in class.
- Cumulative Final is worth 100 points.

Out of the 400 total base points in the course, letter grades will be assigned on the following basis:

- A student who accumulates 352 or more points earns an A.
- A student who accumulates 310 to 351.5 points earns a B.
- A student who accumulates 268 to 309.5 points earns a C.
- A student who accumulates 226 to 267.5 points earns a D.
- A student who accumulates less than 226 points earns an F.

Students are expected to take quizzes and exams on the dates that are given. The format of the quizzes could be either take home or in class. The dates of the quizzes may or may not be announced in advance. A grade of zero, 0, will be given for any missed quiz. Take home quizzes are due at the beginning of the class period on the particular due day. Failure to turn in a take home quiz at the start of the class period will result in a grade of zero, 0. An in class quiz may be given at any time during a class period. No extra time will be given to students who are late for taking the quiz. There will be absolutely NO make up for any missed quiz.

A grade of zero, 0, will be given for any missed examination. If you know that there may be some problem or situation that may prevent you from taking any exam, please let the instructor know in advance. In order to make up a missed exam, a student who misses an exam due to an emergency MUST submit "written" evidence detailing the emergency. The evidence must include a third-party name and contact information (phone number, email address, etc.) in the event verification of the emergency is

needed. It is important to note that a make-up exam may NOT be identical or even comparable in format to the exam originally administered as scheduled. If a student misses an exam without providing acceptable evidence of an emergency, the student will receive a score of zero, 0, for the missed exam. If the instructor accepts the student's excuse, the student must take the make up exam, other than the final, within two days after the student's return. For example, if a student returns on Wednesday and is granted an opportunity to make up for the missed exam, then the student must take the make up exam no later than Friday. A student who completes every other course requirement but misses the final exam due to an emergency and submits acceptable evidence in a timely manner will be assigned a tentative grade of "I" (incomplete) and must take a make up exam during the next semester. Note that it is the student's responsibility to contact the instructor the following semester in order to arrange a time to make up the final exam before the deadline.

Students are expected to check UH Portal and their hawaii.edu email accounts regularly for possible announcements. Students are responsible for downloading take home quizzes and possibly other handouts from UH Portal. In order to access the files on UH Portal, please follow the instructions below:

- Login to your UH Portal account
- Click on My Tools tab
- Make sure that the course registered for the semester are displayed (You may choose the desired semester from the dropdown menu.)
- Click on your course link (Math 140)
- Click on Files from the options
- Select the desired document to download

Note: You need Adobe Reader to open the files.

Student with Disabilities Statement:

Leeward Community College abides by Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990, which stipulate that no student shall be denied the benefits of an education "solely by reason of a handicap." Students with documented disabilities who believe that they may need accommodations in this class are encouraged to contact the Coordinator of the KAKO'O 'IKE (KI) program as soon as possible to ensure that such accommodations are implemented in a timely fashion. The KI office is located in L-208, across from the elevator in the library building or call for information at 455-0421.

Reading and Recommended Problems

Answers to nearly all of these problems are printed in the back of your textbook. Step by step solutions are presented in the Student's Solutions Manual. You will NOT be submitting these problems. This listing is NOT an exhaustive list of everything that you will learn this semester.

You are expected to read the textbook and study the examples presented in the textbook.

The lectures will cover chapter 6, chapter 7, chapter 8, sections 9.1, 9.4, 9.5, 9.6, 11.2, 11.4, and 11.5. Sections 9.2 and 9.3 are optional.

Recommended problems by the sections:

Section	Recommended Problems
6.1	1, 5, 7, 11, 15, 21, 25, 27, 31, 33, 35, 39
6.2	13, 15, 17, 19, 27, 29, 35, 37, 43, 45, 49, 51, 55, 57
6.3	3, 5, 7, 9, 11, 21, 25, 27, 31, 35, 37
6.4	1, 3, 5, 11, 13, 17, 25, 29, 33, 39, 41, 45, 47, 49, 51, 52, 59
6.5	1, 7, 9, 11, 19, 21, 25, 29, 33, 37, 39, 41
7.1	3, 5, 7, 9, 11, 13, 19, 20, 21, 27, 31, 35, 37, 39
7.2	3, 5, 7, 9, 11, 15, 17, 21, 37
7.3	1, 5, 7, 9, 23, 25, 29, 31, 37, 39, 41, 43, 45, 47, 51, 57
7.4	3, 5, 9, 15, 19, 21, 29, 31, 33, 35, 56, 57, 58, 59, 60
7.5	1, 3, 5, 7, 13, 15, 21, 25, 29, 31, 41, 43
7.6	1, 3, 5
7.7	1, 5, 7, 9, 11, 21, 25, 29, 31, 35, 37
8.1	3, 7, 9, 11, 17, 19, 23, 27, 29, 31, 35, 39, 41, 45, 49, 53, 55
8.2	1, 3, 7, 9, 13, 15, 17, 23, 25, 27, 35, 41, 47
8.3	1, 5, 9, 13, 19, 25, 29, 31, 35
8.4	7, 11, 15, 17, 19, 35, 37
8.5	1, 5, 9, 13, 17, 25, 27, 29, 31, 33, 35, 37, 43, 57, 59
9.1	1, 5, 9, 11, 13, 15, 17, 21, 25, 27, 31, 35, 39
9.4	1, 3, 9, 13, 15, 17
9.5	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 27, 29, 31, 41, 43
11.2	1, 3, 13, 19, 23, 25, 29, 31, 33, 37
11.4	1, 7, 13, 15, 19, 21, 23, 25, 27, 29, 31
11.5	1, 3, 11, 15, 19, 21, 23, 27, 29, 31, 33, 35, 39