Foundations Course Articulation Renewal Proposal
Mathematics 205: *Calculus I*
As an Symbolic Reasoning (FS) Course

Leeward Community College
Spring 2008

Page	Contents
2	Course Description & Changes
3	Hallmark 1 Discussion
4	Hallmark 2 Discussion
4-5	Hallmark 3 Discussion
5-6	Hallmark 4 Discussion
6-7	Hallmark 5 Discussion
7	Hallmark 6 Discussion
8-10	Sample Syllabus

I. Course Description.

- Title & Catalog Description

  CALCULUS I (4 credits): Limits, continuity, and derivatives. Computations of derivatives — sum, product, and quotient formulas; implicit differentiation; chain-rule. Study of algebraic and trigonometric functions with analysis of their derivatives. Linear approximation and Newton's method. Applications of derivatives to maximum-minimum problems and related rate problems. Mean-value theorem. Definite integrals and the Fundamental Theorem of Calculus. Applications of definite integrals to computations of areas, volumes, arc lengths, surface area, etc. Simple differential equations. Simpson's rule.

- Student Learning Outcomes

  Upon successful completion of Math 205, a student should:

  - understand and be able to compute limits;
  - understand continuity and be able to identify continuous and discontinuous functions;
  - understand the derivative as a generalization of the rate of change of a linear function;
  - understand the derivation of differentiation formulas (sum, difference, product, quotient, power, etc.);
  - be able to compute derivatives of various functions using the derived differentiation formulas;
  - understand and be able to use the concepts of increasing and decreasing functions, relative and absolute minimums and maximums, and the mean value theorem;
  - be able to solve various application problems involving differentiation, including related rates and minimum/maximum problems;
  - understand and be able to calculate antiderivatives;
  - understand the concept of the definite integral;
  - understand and be able to use the Fundamental Theorem of Calculus;
  - be able to use substitution to calculate integrals;
  - be able to solve various application problems involving integration, including area between curves and volumes using the disk, washer, and shell methods.

II. Changes.

No significant changes have been made in Math 205 since the original request for foundations designation was approved.

III. Assessing the Course: Below are samples of course materials that illustrate how the course meets the Foundations Hallmarks.

Foundations Hallmarks & Application Questions SYMBOLIC REASONING (FS)

1. **Students will be exposed to the beauty, power, clarity and precision of formal system.** ***How will the course meet this hallmark*****?**

   Calculus, separately discovered by Sir Isaac Newton of England and German mathematician Gottfried Wilhelm Leibnitz in late 17^{th} century, is one of the most powerful and influential formal systems and most fruitful strategies ever devised for analyzing our world. The invention of calculus was able to complete the connection between algebra and geometry, which coexisted for over seven hundred years but were not well linked. Calculus, originated in our desire to understand motion, provides the foundation to physics, engineering, and many higher math courses. It is also important to chemistry, astronomy, economics and statistics. Calculus is the mathematical structure that lies at the core of a world of seemingly unrelated issues.

   Math 205 is a traditional introductory calculus course for students majoring in science, technology, engineering and mathematics. Students are introduced to the subtle but very useful concepts of infinity, infinitesimals, and limits. We then apply these concepts to difference quotients and Riemann sums to define the powerful derivatives and integrals. The abstract definitions and theorems enable students to analyze the properties of functions using calculus, while in the applied part of the course they use these ideas to solve problems such as optimization, rate of change, area, and volume. The connection is made by modeling real world phenomena in mathematical terms.

   Sample material:

   i. Prove that $\lim_{x \to -5} \left(4 - \frac{3x}{5} \right) = 7$ using $\varepsilon - \delta$ definition of limit.

   ii. Show that of all the rectangles with a given area, the one with smallest perimeter is a square.

   iii. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5 L/s. This explains, in part, why the function $f(t) = \frac{1}{2}\sin(2\pi t/5)$ has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time *t*.

2. **Instructors will help students understand the concept of proof as a chain of inferences.** ***How will instructors help students understand this concept?***

The proofs of all important theorems and properties are studied or discussed in class, and students are responsible for some definitions, proofs and derivations. For example, the proof of one of the most perfect and beautiful mathematical theorems, the *Fundamental Theorem of Calculus*, is demonstrated in class. In proving the theorem, one needs to use the properties of the integral, which were presented to the students in an earlier section in the same chapter, and also needs to apply the definition of limit of a function, the definition of continuity, the Extreme Value Theorem, the Squeeze Theorem, and the definition of one-sided limits, which were all shown to the students in earlier chapters. The proof of the theorem integrates all the essential concepts in the course up to that point.

Sample material:

i. Prove that $\lim_{x\to 0^+} \sqrt{x}\cos\frac{3}{x} = 0$.

ii. Show that the equation $1+2x+x^3+4x^5=0$ has exactly one real root.

iii. Does there exist a function f such that $f(0)=-1$, $f(2)=4$, and $f'(x)\le 2$ for all x?

3. **Instructors will teach students how to apply formal rules or algorithms.** ***How will instructors meet this hallmark?***

An essential part of calculus has been the development of rules and algorithms. Throughout the semester, students will be able to see how some of the basic concepts, rules, and algorithms may be applied to many fields, and hence begin to understand the power of calculus. Differentiation and integration make up the core of this introductory calculus course. When teaching these two topics, the instructor will demonstrate to students how the formulas or algorithms may be applied through numerous examples. Also, in the chapter of applications of differentiation, the instructor will show students how an algorithmic approach may be used to analyze the graph of a function with many intricate properties.

Sample material:

i. Find $\frac{d^{99}}{dx^{99}}(\sin x)$.

ii. Evaluate $\int_0^{\pi/2} \frac{3\sin x\cos x}{\sqrt{1+3\sin^2 x}}\,dx$.

iii. Given that $f(x)=\dfrac{x^2}{(x-2)^2}$, $f'(x)=\dfrac{-4x}{(x-2)^3}$, and $f''(x)=\dfrac{8(x+1)}{(x-2)^4}$, answer the following questions.

a. Domain
b. *x*-intercept(s)
c. *y*-intercept
d. Vertical asymptote(s)
e. Horizontal asymptote(s)
f. Critical point(s)
g. Interval(s) of increase
h. Interval(s) of decrease
i. Local extreme value(s) and where
j. Interval(s) of concave up
k. Interval(s) of concave down
l. Inflection point(s)
m. Sketch the graph

4. **Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence. What symbolic techniques will be required and in what contexts?** ***How will presentations and evaluations be incorporated into the course?***

Students will routinely have to interpret symbolic statements, and to appropriately use symbols in their homework, quizzes, and exams. They are introduced to the symbols of calculus throughout the semester, and then their skills are improved through practice and feedback. For example, in a typical application problem, a student takes a real world problem and reduces it to an abstract symbolic one that is formulated in mathematical language. The student then solves the problem abstractly, by applying the calculus to the model. Finally, the student translates the mathematical solution back into the terms of the given problem and verifies the validity of the solution.

Sample material:

i. A crystal growth furnace is used in research to determine how best to manufacture crystals used in electronic components for the space shuttle. For proper growth of the crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by $T(w)=0.1w^2+2.155w+20$, where T is the temperature in degrees Celsius and w is the power input in watts.

a. How much power is needed to maintain the temperature at $200^\circ C$? Round your answer to 3 decimal places.

b. If the temperature is allowed to vary from $200^\circ C$ by up to $\pm 1^\circ C$, what range of wattage is allowed for the input power? Round your answer to 3 decimal places.

c. In terms of the ε, δ definition of $\lim_{x \to a} f(x) = L$, what is *x*? What is $f(x)$? What is *a*? What is *L*? What value of ε is given? What is the corresponding value of δ?

ii. Express the $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$ as a definite integral.

iii. Find $\frac{dy}{dx}$ for $y = \int_{\tan x}^{0} \frac{dt}{1+t^2}$.

5. **The course will not focus solely on computational skills.** ***What reasoning skills will be taught in the course?***

Being developed through logic and reason, calculus is naturally applied to problems that require a high level of reasoning ability. First, the instructor will present arguments and proofs on a regular basis that illustrate sophisticated reasoning. The instructor can choose from a wealth of theorems and facts that have very logical, and sometimes clever proofs. Secondly, the students are required to supply their own arguments for homework, quizzes or exams. They must supply the logical form of the arguments, incorporate the details that are presented, decide which theorems are applicable or sufficient, and present the results in a readable and cogent manner.

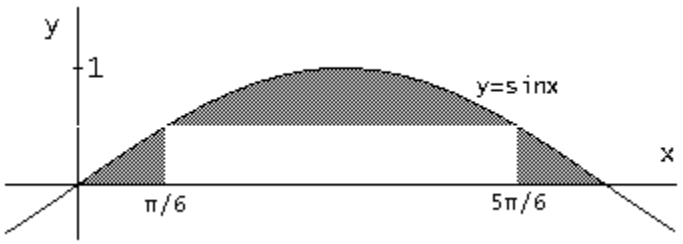
The types of problems that require such reasoning skills can be found at almost any point in the course, but especially in the following areas: analyzing functions, applying Fermat's Theorem, the Intermediate Value Theorem, and the Fundamental Theorem of Calculus, and almost any type of applied problem.

Sample material:

i. Suppose that f is differentiable on $(-\infty, \infty)$ and has two roots. Show that f' has at least one real root.

ii. Find f if $f''(x) = \frac{1}{x^3}, x > 0, f(1) = 0, f(2) = 0$.

iii. Find the area of the shaded region.



6. **Instructors will build a bridge from theory to practice and show students how to traverse this bridge.** ***How will instructors help students make connections between theory and practice?***

   The new material is developed concurrently with applications or visual presentations whenever possible. For example, the limit of a function is developed with the discussion of a falling object, with the instantaneous velocity being the limit of the average velocities. Students can then refer to this example as a touchstone when further interpreting limits. With the theory of differential calculus, the theoretical results can be applied to a variety of applications, including linear approximations and maximization problems, but the theory is most fully utilized when analyzing functions and curve sketching. The instructor will present examples and models for the students to investigate, analyze, pattern and extrapolate. Students' learning will be practiced and demonstrated throughout the semester via quizzes, homework, in-class participation and exams.

   Sample material:

   i. If a ball is thrown into the air with a velocity of 50ft/s, its height in feet *t* seconds later is given by $y = 50t - 16t^2$. Find the average velocity for the time period beginning when t = 2 and lasting

      a. 0.5 seconds
      b. 0.1 second
      c. 0.05 second
      d. 0.01 second.
      e. Find the instantaneous velocity when t = 2.

   ii. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

   iii. A particle moves along a line so that its velocity at time *t* is $v(t) = t^2 - t - 6$ measured in meters per second. Find the displacement and the distance traveled during the time period $1 \le t \le 4$.

Sample Syllabus

LEEWARD COMMUNITY COLLEGE
Mathematics and Natural Sciences Division
Course Syllabus
Math 205 – Calculus I (4.0 credits)

Instructor: **Office Hours**: **Office Location**:
Phone: **E-mail**:

Catalog Course Description: Limits, continuity, and derivatives. Computations of derivatives — sum, product, and quotient formulas; implicit differentiation; chain-rule. Study of algebraic and trigonometric functions with the analysis of their derivatives. Linear approximation and Newton's method. Applications of derivatives to maximum-minimum problems and related rate problems. Mean- value theorem. Definite integrals and the Fundamental Theorem of Calculus. Applications of definite integrals to computations of areas, volumes, arc lengths, surface areas, etc. Simple differential equations. Simpson's rule.

Co-requisites: None
Prerequisites: C or better in MATH 140 or in equivalent courses covering trigonometry and analytic geometry
Recommended Preparations: None

Textbook and other Resources:

- Essential Calculus with Differential Equations by James Stewart, custom made for University of Hawaii Leeward Community College.

Student Learning Outcomes:

Upon successful completion of Math 205, a student should:

- understand and be able to compute limits;
- understand continuity and be able to identify continuous and discontinuous functions;
- understand the derivative as a generalization of the rate of change of a linear function;
- understand the derivation of differentiation formulas (sum, difference, product, quotient, power, etc.);
- be able to compute derivatives of various functions using the derived differentiation formulas;
- understand and be able to use the concepts of increasing and decreasing functions, relative and absolute minimums and maximums, and the mean value theorem;
- be able to solve various application problems involving differentiation, including related rates and minimum/maximum problems;

- understand and be able to calculate antiderivatives;
- understand the concept of the definite integral.
- understand and be able to use the Fundamental Theorem of Calculus.
- be able to use substitution to calculate integrals.
- be able to solve various application problems involving integration, including area between curves and volumes using the disk, washer, and shell methods.

Academic Calendar:

Attendance: Students are expected to attend each class session ON TIME. They are also expected to stay for the entire duration of the class. Anticipated as well as unexpected absences should be discussed with the instructor. The acceptance of an excuse for absence is at the discretion of the instructor. Students are expected to have their textbooks and note-taking equipment (writing instrument and paper) during each class session. Beyond being physically present in the room students are expected to be alert and engaged in the lecture or discussion. Students will be responsible for all material, discussion, and assignments covered during any missed class session(s). Students who miss class (for whatever reason) should arrange to obtain a set of class notes from a classmate, read through the covered sections and attempt the assigned problems. Students who miss a class will NOT be provided with a repeat of the missed lecture.

Grading Policy: Course evaluation will be done through assignments, quizzes and examinations.

- Assignments and quizzes: 100 points.
- Two Midterms, each is worth 100 points, a total of 200 points. Exam dates will be announced in class.
- Cumulative Final is worth 100 points.

Out of the 400 total base points in the course, letter grades will be assigned on the following basis:

- A student who accumulates 352 or more points earns an A.
- A student who accumulates 310 to 351.5 points earns a B.
- A student who accumulates 268 to 309.5 points earns a C.
- A student who accumulates 226 to 267.5 points earns a D.
- A student who accumulates less than 226 points earns an F.

Special Note:

- Each assignment is due at the beginning of the class period on the day it is due. Assignments turned in more than 5 minutes late will not be accepted.

- The dates of the in class quizzes may or may not be announced in advance. Quizzes may be given at any time during a class period. NO extra time will be given to students who are late for taking the quiz.
- Absolutely NO make up for any missed quizzes or assignments.
- A student who misses an exam due to an emergency MUST submit "written" evidence detailing the emergency in order to make up. The evidence must include a third-party name and contact information (phone number, email address, etc.) in the event verification of the emergency is needed. The acceptance of an excuse is at the discretion of the instructor. A make-up exam may NOT be identical or even comparable in format to the exam originally administered as scheduled. A make-up exam, other than the final, must be taken within two days upon the student's return.
- A student who completes every other course requirement but misses the final exam due to an emergency and submits acceptable evidence in a timely manner will be assigned a tentative grade of "I" (incomplete) and must take a make up exam during the next semester. Note that it is the student's responsibility to contact the instructor the following semester in order to arrange a time to make up the final exam before the deadline.

Students are expected to check UH Portal and their hawaii.edu email accounts regularly for possible announcements. Students are responsible for downloading take home quizzes and possibly other handouts from UH Portal. In order to access the files on UH Portal, please follow the instructions below:

- Login your UH Portal account
- Click on My Tools tab
- Make sure that the course registered for are displayed (You may choose the desired semester from the dropdown menu.)
- Click on your course link (Math 205)
- Click on Files from the options
- Select the desired document to download

Note: You need Adobe Reader to open the files. Depending on the security set up on your computer, you may have to save the file first before viewing/opening the file.

Student with Disabilities Statement:

Leeward Community College abides by Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990, which stipulate that no student shall be denied the benefits of an education "solely by reason of a handicap." Students with documented disabilities who believe that they may need accommodations in this class are encouraged to contact the Coordinator of the KAKOʻO ʻIKE (KI) program as soon as possible to ensure that such accommodations are implemented in a timely fashion. The KI office is located in L-208, across from the elevator in the library building or call for information at 455-0421.