Request for Renewal of
MATH 112: Math for Elementary Teachers II
as an FS course

Leeward Community College
Spring 2011

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MATH 112: Math for Elementary Teachers II (3)
AA/FS
3 hours of lecture per week

Course Description

MATH 112 is the second of a year-long sequence (MATH 111 - MATH 112) designed to provide a rigorous background in mathematical concepts and reasoning for students intending careers in Elementary Education. The emphasis is on understanding, representing and communicating mathematical ideas, problem solving and reasoning, and constructing and writing elementary proofs. Topics covered over the year include operations (both standard and nonstandard arithmetic) and their properties, ordered n-tuples and their practical applications, set theory, counting, introduction to measurement, patterns and algebra.

Student Learning Outcome

Upon successful completion of MATH 112, a student should, for material covered during the semester, be able to

1) Choose and demonstrate appropriate strategies to investigate and/or solve theoretical and applied problems.
2) Evaluate and analyze mathematical concepts and properties.
3) Use proper representations to describe patterns and relationships.
4) Formulate, articulate and test conjectures.
5) Apply fundamental rules of logic to construct justifications/elementary proofs.
6) Select and utilize precise mathematical language and symbols to effectively communicate procedures and results.

Changes

There have been no substantive changes to MATH 112 since the initial request for Foundations Designation has been approved. However, significant developments at both the campus and system levels required that the Curriculum Central Core Outline be revised. Although course SLOs were rewritten to conform to the guidelines set by the College Assessment Team, the course itself was not modified to ensure that system-wide guidelines for rigor and consistency were adhered to.

Assessment

Samples of course materials that illustrate how the course meets the Foundations Hallmarks are embedded in the descriptions that follow.

Hallmark 1: Students will be exposed to the beauty, power, clarity and precision of formal systems.

The natural numbers, the integers, the rationals and the reals, each with its standard operations, are excellent examples of formal systems. Either they will be introduced axiomatically, as is the case for the natural numbers, or they will be introduced to model a variety of situations that occur, such as in business, physics or geometry that have certain common intrinsic characteristics. We will use the power, clarity and precision of these formal systems to establish many of their properties. Most importantly, the clarity and precision of these systems will open them to very interesting comparisons, which allows us to use what we learn about one to understand another more easily; this more than anything displays the power of formal systems. The natural numbers with their operations will be built on a couple of axioms about counting the elements of a set; if there is beauty in simplicity then the students will see this kind of beauty as we define and examine the four standard operations on the natural numbers.

Sample Course Material

Students are consistently exposed to the power and beauty of mathematics as they engage in discussions to deepen their understanding of the various number systems. As they formulate their responses to specific questions like the ones that follow, they strengthen their awareness of the clarity and precision of such formal mathematical systems.

1. Here are some questions to test your understanding of subtraction.
   a. Explain why $9 - 5 = 4$ in two ways. Use *the Definition of Subtraction* and *Subtraction Using Sets.*
   b. Explain why $n - n = 0$ for every natural number n. Conversely, show that if $m - n = 0$, then $m = n$.
   c. Explain why $(l + m) - m = l$ for every natural numbers m and n.

2. Compare and contrast the presentation of addition and subtraction to the presentation of multiplication and division. In what ways are addition and multiplication alike; in what ways are subtraction and division alike? How is the relation of subtraction to addition like the relation of division to multiplication? How are these operations and their relationship to one another different?

Hallmark 2: Instructors will help students understand the concept of proof as a chain of inferences.

The instructors will present proofs from time to time, e.g. that there exist infinitely many primes or that there exist numbers besides the rationals. More importantly, the students will discover and write proofs. Often this will take the form of the students presenting a conjecture followed by the struggle to see if the conjecture is indeed true; if the conjecture is discovered to be false the student will be expected to provide a proof that confirms that; if the conjecture is true, then the student will be expected to provide a proof that confirms that. From time to time the students will be asked to use previous "more basic," results to establish more sophisticated results. This will require the presentation of a proof.

Sample Course Material

On a daily basis, students are expected to construct formal proofs of varying levels of complexity. The three written assignments that follow are typical.

1. Let set A be the set of all subsets of the empty set. Use the definition of a finite set to justify that set A is finite.

2. Prove that addition of natural numbers is associative.

3. Prove: If a is divisible by b and m is divisible by n, then $\left(\frac{a}{b}\right)\left(\frac{m}{n}\right)=\frac{(am)}{(bn)}$.

Hallmark 3. Instructors will teach students how to apply formal rules or algorithms.

An integral part of any mathematics course is the collection of algorithms as well as the formal rules that govern the manner in which these algorithms are applied to make sense of conceptual and abstract situations as well as real-world applications. Throughout the course, instructors emphasize the analytical processes used to select the appropriate algorithms and the correct implementation of said algorithms. In addition, instructors teach and demonstrate critical strategies of recognizing the correctness or reasonableness of results obtained.

Sample Course Material

1. In planning to write up the formal proof required in the following problem, the students are required to use the formal rules and algorithms of set operations.

Let P and Q be any two sets. Prove that the following statement holds.

$$\#(P \cup Q) = \#(P \setminus Q) + \#(Q \setminus P) + \#(P \cap Q)$$

2. Fundamental concepts relating to conditional statements are used to analyze general principles. The following are typical course questions.

Given below is a conditional statement.

If it is raining, then the ground is wet.

a. Write the hypothesis of the statement.
b. Write the conclusion of the statement.
c. Write the converse of the statement.
d. Write the contrapositive of the statement.

Consider the following statement.

If x is an element of A, then x is an element of $A \cup B$.

a. Describe the characteristics of an example that would show that the statement is false.
b. Write the first statement of a justification that the statement is true.
c. Write the last statement of a justification that the statement is true.

Write the two conditional statements that are implied by the following definition of a subset.

Let M and P be any two sets. Set M is a subset of set P if every member of set M is a member of set P.

Explain the two uses of the definition of equality of two sets A and B.

Hallmark 4. Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence.

In this course, students formulate new mathematical knowledge primarily through problem-solving. Various problem solving strategies will be explored and different mathematical representations to model and solve problems will be demonstrated. One constant refrain of the course is to state anything of consequence both verbally and symbolically.

Sample Course Material

The study of the natural numbers with their operations will be built on the formal axioms about the number of elements of a set. Such approach requires that the students are very proficient at defining, both symbolically and verbally, the rule that sets up the equivalence between two sets. Sample questions follow.

1. Consider set S = { {}, {(a, b)}, {x, y}, {a, e, i, o, u}, {c, a, t}, {t, e, e, n, y} }. There is a set N whose elements can be systematically paired with the elements of set S according to the following pairing description

p ↔ number of elements in p , where p is an element of set S.

a. Write out explicitly the pairing described.
b. Use standard notation to display set N.

2. Let set A = { a, b, c, d, e } and set B = { {a}, {b}, {c}, {d}, {e} }. Determine a systematic pairing between the elements of A and the elements of B.

a. Present an explicit description of that pairing.
b. Write a symbolic description of that pairing.

Typical problems that require students to use symbolic techniques to present arguments regarding number of elements of sets are shown below.

1. Maggie was hired by an advertising company to interview people to find out what brand of ice cream they liked. She submitted the following report.

Number of people interviewed	100
Number of people who liked Creamy Delight	78
Number of people who liked Sweet Treat	65
Number of people who liked both Creamy Delight and Sweet Treat	40

Every person interviewed liked one or the other brand of ice cream.

After the supervisor read Maggie's report, he fired Maggie. Why?

2. Common ailments of senior citizens are arthritis, arteriosclerosis and loss of hearing. A survey of 200 residents at Golden Homes found that 70 had arthritis, 60 had arteriosclerosis, 80 had loss of hearing, 35 had arthritis and arteriosclerosis, 33 had arthritis and loss of hearing, 31 had arteriosclerosis and loss of hearing and 15 had all three.

a. Construct a Venn diagram to illustrate the above survey data/information.
b. Of those surveyed, how many had
   none of the three ailments? _____
   arthritis but neither of the other two? _____
   exactly one of those ailments? _____
   arteriosclerosis and loss of hearing but not arthritis? _____
   arteriosclerosis or loss of hearing but not arthritis? _____
   exactly two of these ailments? _____

Hallmark 5. The course will not focus solely on computational skills.

As dictated by the course description, Math 112 does not focus on computational skills. Logical thinking and reasoning are paramount. Although standard operations on the different sets of numbers are studied, the emphasis is on understanding the properties that govern these operations.

Students will be asked to infer from a collection of examples what may be true in general. Students will learn techniques to refute some conjectures and use deductive methods of proof to establish that other conjectures are true. Proof by contradiction will be presented and students will be asked to write a few proofs which require that technique. While inductive proofs will not be presented or worked with formally, students will be encouraged to use iterative methods where appropriate "to establish" that some formula is true in general.

Sample Course Material

1. The class session that is designed to help the students formulate a definition of addition of natural numbers is usually conducted by having students work in groups to discuss the common characteristics of several questions that can be answered by performing additions of numbers. Because the focus of the discussion is on the characteristics of the questions, students are not focusing on the actual computation results.
2. The question below shows that the emphasis is not on computation.

   Mr. and Mrs. Potter have three children, Peter, Paul and Mary. Each of their children throw pots, i.e., makes pots out of clay on a wheel. Last week each child made five pots; as they made a pot they marked it with the number 1, 2, 3, 4, or 5 to keep track of the order in which they were made. Show that altogether Mr. and Mrs. Potter's children made 3 x 5 pots by pairing the elements of the set of all pots they made last week with the elements of a set of the form A x B, for some appropriate choices of sets A and B. (You are not being asked to show that there are 15 pots. You are being asked to show that there are 3 x 5 pots by using the only thing you know about multiplication, the Definition of Multiplication.)
3. An example of a class assignment on proof by contradiction follows.

   Use proof by contradiction to prove that the sets H = {1, 2, 3, . . . , 1000000000000} and G = {1, 2, 3, . . . , 1000000000003} are not equivalent.

Hallmark 6. Instructors will build a bridge from theory to practice and show students how to traverse this bridge.

Everyday situations that are familiar to the students are often used to motivate the discussion of the abstract mathematical concepts. At other times, real applications are examined only after the axiomatic model has been discussed.

Sample Course Material

1. Students are asked to examine the descriptions of actual situations to help them formulate the abstract definitions for the operations on natural numbers. The following is a typical one for multiplication.

Study the following situations and describe the common characteristics.

a. Cody likes to make up passwords for his family and friends. His baby brother knows how to read and write the first 4 letters of the alphabet and seems to be able to remember any two symbols in succession. Cody wonders, "How many 2 letter passwords can one make using those 4 letters?"

b. Alice loves to bake and wants to make 4 cakes for a party. She needs 5 teaspoons of vanilla for each cake. She wants to know how many teaspoons of vanilla she will need for all the cakes.

c. Will and Tracy have been asked to set up some chairs for a performance. They decide that for the room they will use they can set up 5 rows of chairs with 6 chairs in each row. They need to know how many chairs to order.

d. Pamela is an electrician. She carries a lot of different kinds of wire on her truck. The wires she carries come in four different colors, red, black, green and white. Each color of wire comes in three different gauges, 25, 15 and 10. (The gauge of a wire indicates its thickness.) How many different kinds of wire does Pamela carry in her truck?

2. Based on an axiomatic development we discover and prove a variety of properties and symmetries for the four standard operations. Generally, each of these properties or symmetries has some explicit utilities in the computational realm. For example, there are a variety of tricks for carrying out certain kinds of subtraction and division problems that take advantage of a property of symmetry to change the given problem into a much simpler problem of the same kind with the same result. One typical group discussion question on using symmetry properties of subtraction follows.

Subtraction has a property, which can be referred to as "maintaining the difference" symmetry.

a. State the property symbolically.
b. State the property in words.
c. Explain how the property can be applied to complete the following subtractions.
   401 – 29, 81 – 49, 4010 - 2994

LEEWARD COMMUNITY COLLEGE
Mathematics and Natural Sciences Division
Course Syllabus - SPRING 2009

MATH 112 - MATHFOR ELEMENTARY TEACHERS II (3 hours credits)

Instructor:
Office Hours:
Office Location:
Contact Information:

CATALOG COURSE DESCRIPTION: Math 112 covers representations of and operations on natural numbers, integers, rationals, and reals, and the properties of those operations. There will be connections to other parts of mathematics and applications.

CO-REQUISITES: None
PREREQUISITES: Math 111 with C or better, within the past two years
RECOMMENDED PREPARATIONS: None

TEXTBOOKS AND OTHER RESOURCES: Mathematics for Educators, Joel L. Weiner

STUDENT LEARNING OUTCOMES: Upon successful completion of Math 112, the student should be able to

- use various mathematical reasoning and problem solving strategies to investigate and understand problems involving operations, natural numbers, integers, rationals and reals
- comprehend both oral and written communication of appropriate mathematical concepts that are presented
- communicate with words and also in combination of words and symbols the appropriate mathematical concepts and procedures
- use certain manipulative and teaching aids to explore mathematical concepts and procedures
- describe objects, procedures, patterns, and relationships with clarity and precision using various representations
- extend patterns and make conjectures relating to appropriate mathematical relationships and applications

ATTENDANCE: Students are expected to attend each class session. They are also expected to be on time and to stay for the entire duration of the class. Therefore, repeatedly missing any portion of a class session may adversely affect student's learning. If a student misses any part of a class session, he/she is responsible for taking the necessary steps to learn the course material in a timely manner.

HOMEWORK: Completion of homework is critical to student success. Class discussions and activities require students to prepare by reading and answering questions from the text or supplemental course materials before they come to class. Students should complete assigned activities within the timeframe set.

CLASS ACTIVITIES/DISCUSIONS: A significant part of class time will be devoted to the process of doing mathematics. Students will be expected to engage in various activities designed to make them better readers and writers of mathematics as well as better problem solvers. All activities must be completed within the timeframe set.

GRADING POLICY: The final course grade will be based on accumulated points from examinations, quizzes, homework and class activities/discussion. Letter grades will be assigned as follows:

90% - 100% of possible points --- A
80% - 89% of possible points ---- B
70% - 79% of possible points ---- C
60% - 69% of possible points ---- D
below 60% of possible points ---- F

The class has a ‘no make-up’ policy. Late work will not be accepted and missing work will be graded as a zero.
A letter grade of 'C' is considered the minimum level of achievement for subsequent study of higher mathematics.
A student must formally withdraw (no later than March 23, 2009) from the course to be assigned a 'W' grade.

STUDENT WITH DISABILITIES STATEMENT: Leeward Community College abides by Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990, which stipulate that no student shall be denied the benefits of an education "solely by reason of a handicap." Students with documented disabilities who believe that they may need accommodations in this class are encouraged to contact the Coordinator of the KAKO‘O ‘IKE (KI) program as soon as possible to ensure that such accommodations are implemented in a timely fashion. The KI office is located in L-208, across from the elevator in the library building or call for information at 455-0421.