Foundations Course Articulation New Proposal
Mathematics 135: *Pre-Calculus: Elementary Functions*
As an Symbolic Reasoning (FS) Course

Leeward Community College
Spring 2008

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I. Course Description.

- <u>Title & Catalog Description</u>

  A functional approach to algebra which includes polynomial, exponential and logarithmic functions; higher degree equations; inequalities; sequences; binomial theorem; partial fractions. This course is recommended for students majoring in mathematics, sciences or engineering.

- <u>Student Learning Outcomes</u>

  Upon successful completion of Math 135, a student should:

  1. Understand and be able to solve equations and inequalities, including those involving radical, exponential, and logarithmic expressions.
  2. Understand and be able to analyze, combine, and find the inverses of functions.
  3. Be able to use properties to construct graphs of relations and functions in the Cartesian plane, including the conic sections.
  4. Understand and be able to analyze the properties of polynomial functions and their graphs, including symmetry, intercepts, and zeros with their multiplicities.
  5. Be able to analyze the properties of rational functions and their graphs, including domain, range, asymptotes, and intercepts.
  6. Understand and be able to analyze the properties of exponential and logarithmic functions and their graphs, including domain, range, and asymptotes.
  7. Be able to model and solve various applications problems related to the studied relations and functions

II. Changes.

This is the original request for symbolic reasoning (FS) designation.

III. Assessing the Course: Below are samples of course materials that illustrate how the course meets the Foundations Hallmarks.

<u>Foundations Hallmarks & Application Questions SYMBOLIC REASONING (FS)</u>

1. **Students will be exposed to the beauty, power, clarity and precision of formal system. *How will the course meet this hallmark*?**

Math 135 is a standard one-semester precalculus course on elementary functions. The concept of functions is one of the most useful and broad-ranging ideas in all of mathematics. Functions can be represented in algebraic form using an equation or formula, verbal form, tabular form using a table, and graphical form using a coordinate system and graph. These forms provide the instructor with the opportunity to demonstrate how useful the concept of function applies to numerous mathematical, business and science applications. For example, functions are used to compute the average rate of increase or decrease of a patient's temperature over given time periods. Functions are also used to analyze and compare changing patterns of consumption of red meat in China and the United States. It is also used to determine the rate of spread of an oil spill caused by a leak from an offshore oil rig.

Throughout the course, the basic techniques of graphing will help students understand the features of a graph. For the graphing of non-linear functions such as polynomial, rational, exponential, and logarithmic functions, students will see the beauty of these graphs when they study the concepts of symmetry, asymptotes, translations, and reflections. Along with these properties, students will be shown the power of analyzing in new contexts some familiar procedures, including finding the intercepts and zeros, domain, range, and approximating coordinates of the turning points to find maximum and minimum values. These aspects of graphs are compared to tabular and symbolic representations of the same functions, which illustrates the precision, power, and clarity of having distinct but related formal systems of studying functions.

In the chapter on Polynomial and Rational Functions, students will use their knowledge of functions as a powerful tool to solve real world applications. For example, the functions and ideas introduced in this chapter will help in modeling data relating smoking and lung cancer, summarizing global trends in airline passenger miles using a quadratic model, modeling the spread of AIDS using linear and quadratic functions and determining long-term behavior by computing the horizontal asymptote of a rational function.

In the section on Maximum and Minimum Problems, the discussion begins on a more intuitive level in trying to sort through an infinite number of possible cases and pick out the required maximum or minimum value for this situation. In these exercises there are many pairs of possibilities and the students are expected to apply the knowledge of quadratic functions to solve the problem in a precise manner. These applications demonstrate the power of mathematics that will help the students understand and solve a wide range of problems.

The following are sample material extracted from class exercises and homework problems.

i. (a). Using a graphing utility to graph the following three functions on the same set of axes.

$$F(x) = \sqrt[3]{x^3 + 3} - 1$$

$G(x) = \sqrt[3]{x^3+1} - 3$

$H(x) = \sqrt[3]{(x+1)^3 - 3}$

(b). Which two graphs appear to be symmetric about the line y = x? Use algebra to verify that the two functions are indeed inverses.

ii. Draw a regular hexagon on a rectangular coordinate system. Try to find a way to draw the hexagon on another rectangular coordinate system so that figure is

a. symmetric about the y-axis, but not about the x-axis;
b. symmetric about the x-axis, but not about the y-axis;
c. symmetric about the origin and both axes;
d. symmetric about the origin but not about either axis.

Note that in some cases there are many ways, in others none.

iii. The following uses rational functions to model bacteria growth.

A group of agricultural scientists has been studying how the growth of a particular type of bacteria is affected by the acidity level of the soil. One colony of the bacterial is placed in a soil that is slightly acidic. A second colony of the same size is placed in neutral soil. Suppose that after analyzing the data, the scientists determine that the size of each population over time can be modeled by the following functions.

Colony in neutral soil: $y = (2t + 1)/(t + 1)$ $t \geq 0$

Colony in acidic soil: $y = (4t + 3)/(t^2 + 3)$ $t \geq 0$

In both cases, y represents the population, in thousands, after t hours. Find the horizontal asymptotes for each graph and thereby determine the long-term behavior of each colony.

Note: When the student finds the horizontal asymptote for each graph, the colony in the neutral soil will find that with a horizontal asymptote of y = 2, this colony will approach a population of 2,000. For the colony in acidic soil, the horizontal asymptote is y = 0 and the population approaches extinction.

2. **Instructors will help students understand the concept of proof as a chain of inferences.** ***How will instructors help students understand this concept?***

Proofs of important theorems and properties are presented and discussed in class. Students will learn the basics proof techniques of deductive reasoning. They will check to see whether a given situation fits the hypotheses of a procedure and then justify each step in the process. Instructors will help students identify the inferences and techniques that are needed to prove the theorem. For example, to prove the Remainder Theorem, the students will be first exposed to particular cases of the theorem. They will be given the problem of dividing the polynomial $F(x) = 2x^2 - 3x + 4$ by $x - 1$. Then according to the Remainder Theorem, the remainder should be the number F(1). The students will check this by synthetic division. As a second example, they will take the function $g(x) = ax^2 + bx + c$ and divide it by $x - r$. Again, according to the Remainder Theorem, the remainder is equal to g(r). They will check the results by synthetic division. Next, a general proof of the Remainder Theorem can be given along these same lines.

Students will learn that the chain of inferences used to prove the Remainder Theorem may help prove another theorem. The Reminder Theorem will used to prove the Factor Theorem. Similarly, other linear factor theorems are proved with the use of the Factor Theorem.

Other theorems are proved by examining examples. For example, to prove the Rational Roots Theorem, the student will learn that if we suppose that the two rational numbers a/b and c/d are the roots of a certain quadratic equation, then from their experience with quadratic equations, they know that the equation can be written in the from $k(x – a/b)(x – c/d) = 0$, where k is a constant. They will check this by carrying out the multiplication and clearing of the fractions, the equation is $(kbd)x^2 – (kad + kbc)x + kac = 0$. They will observe that a and c are factors of the kac and b and d are factors of the coefficient of x^2 term which the theorem asserts.

In addition to including such deductive proofs in class presentations, instructors will also require that the students prove similar properties as exercises or as quiz/exam problems.

The following are sample materials extracted from class exercises and homework problems.

i. This exercise outlines a proof of the rational roots theorem. At one point in the proof, we will need to rely on the following fact, which is proved in courses in number theory. Fact from Number Theory Suppose that A, B, and C are integers and that A is a factor of the number BC. If A has no factor in common with C(other than $\pm$ 1), then A must be a factor of B.
a. Let A = 2, B = 8, and C = 5. Verify that the fact from number theory is correct here.
b. Let A = 20, B = 8, and C = 5. Note that A is a factor of BC, but A is not a factor of B. Why doesn't this contradict the fact from number theory?
c. Now we are ready to prove the rational roots theorem. We begin with a polynomial equation with integer coefficients:

$$a_nx^n + a_{n-1}x^{n-1} + . . . + a_1x + a_0x = 0 \ (n \geq 1, a_nx \neq 0)$$

We assume that the rational number p/q is a root of the equation and that p and q have no common factors other than 1. Why is the following equation now true?

$$\mathbf{a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + ... + a_1\left(\frac{p}{q}\right)a_o = 0}$$

d. Show that the last equation in part © can be written

$$p(a_np^{n-1} + a_{n-1}qp^{n-2} + . . . + a_1q^{n-1}) = -a_0q^n$$

Since p is a factor of the left-hand side, of this equation, p must also be a factor of the right-hand side. That is, p must be a factor of a_0q^n. But since p and q have no common factors, neither do p and q^n. Our fact from number theory now tells us that p must be a factor of a_0 as we wished to show. (The proof that q is a factor of a_n is carried out in a similar manner.)

ii. Let r_1 , r_2 , r_3 , r_4 be roots of the equation $x^3 + bx^2 + cx + d = 0$. Use the method shown in Example 5 to prove the following facts.

$$r_1 + r_2 + r_3 = - b$$

$r_1r_2 + r_2r_3 + r_3r_1 = c$
$r_1r_2r_3 = -d$

iii. (a). Use base 10 logarithms to solve the equation $2^x - 3 = 0$.
(b). Prove that (log 3)/(log 2) = (ln 3)/(ln2).
(c). Prove that $\log_b (P/Q) = \log_b P - \log_b Q$
Hint: Study the proof of the Product Rule for Logarithms.
(d). Prove the Change-of-Base Formula

$$\log_a x = \frac{\log_b x}{\log_b a}$$

iv. A function f is said to be even if the equation f(-x) = f(x) is satisfied by all values of x in the domain of f.
a. Explain why the graph of an even function must be symmetric about the y-axis.
b. Show that each function is even by computing f(-x) and then noting that f(x) and f(-x) are equal.
(i). $f(x) = x^2$
(ii). $f(x) = 2x^4 - 6$

3. **Instructors will teach students how to apply formal rules or algorithms.** ***How will instructors meet this hallmark?***

The instructors will help the students apply the theorems and rules in a structured format. Students will learn the definitions, properties, and theorems, then apply concrete as well as abstract problems. The following is an example of how this is done. In the study of Roots of Polynomial Equations, students will be given the instructions below.

To find all rational roots of a polynomial equation, the following are the first few steps that will be used.

i. Is the polynomial factorable by grouping? It yes, factor and find all roots.
ii. Check the number of positive and negative roots by using Descartes's Rule of Signs.
iii. If the polynomial is not factorable, list all possible rational roots of the equation.
iv. Test the smallest positive integer in this list to find an upper bound.
v. If an upper bound is found, delete all larger numbers in the list, then test the smallest negative integer to find a lower bound.

The instructor will help students identify the procedures, rules and theorems that are used to find these rational roots. The Descartes's Rule of Signs will be discussed to determine the number of possible positive and negative roots there are. Next the instructor will show how the Rational Roots Theorem can be used limit the number of possibilities of roots to be tested. The Upper and Lower Bound Theorem will be used to tell the student how synthetic division can be used in determining upper and lower bounds for roots. Students will use synthetic division to find upper and lower bounds and to test whether any of the possible rational roots are the actual roots. The instructor will continue in this way to help students to reason why and apply the procedures, rules and theorems to find all the rational roots. The students will then be able to select and apply the appropriate procedure, rules and theorems to complete problems on their own.

The instructor will then help students to learn the concepts through exercises such as the ones shown below.

i. Find the rational roots of the equation, and then solve the equation. (Use the Rational Roots Theorem and the Upper and Lower Bound Theorem.)
   $4x^3 – 10x^2 – 25x + 4 = 0$

ii. The Location Theorem asserts that the polynomial equation f(x) = 0 has a root in the open interval (a, b) whenever f(a) and f(b) have the same sign, can the equation f(x) = 0 have a root between a and b? Hint: Look at the graph of $F(x) = x^2 – 2x + 1$ with a = 0 and b = 2.

The following are problems in other areas that require students to apply formal rules or algorithms.

i.(a).Use the graphing utility to graph both $g(x) = \sqrt[3]{2-x^3}$ and y = x on the same axes.
(b). What kind of symmetry do you observe? What does this tell you about the inverse for g?
(c). Starting with the equation $g(x) = \sqrt[3]{2-x^3}$, follow the two-step procedure to find $g^{-1}(x)$. What do you observe?

ii.(a). Verify the Product Rule for Logarithms using the values b = 10, P = 5, and $Q=\sqrt{2}$.
(b). Let P = 3 and Q = 4. Show that $\ln (P + Q) \neq \ln P + \ln Q$.
(c). Verify the Quotient Rule for Logarithms using the values b = 10, P = 2, and Q = 3.

iii. Graph the rational function. Find its domain, the *x*- and *y*-intercepts, vertical asymptotes and horizontal asymptote. Note that the graph crosses its horizontal asymptote once. Find this point.

4. **Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence. What symbolic techniques will be required and in what contexts?** ***How will presentations and evaluations be incorporated into the course?***

Students will be required to state a concept both verbally and symbolically. They will be taught to formulate a symbolic representation of a concept and use that symbolic representation to explain the ideas represented. Setting up an equation that defines a function is critical in exercises that involve maximum and minimum problems. It is possible to approach these problems with the appropriate symbolic techniques in a systematic manner. For example, the following procedure will be presented to students to set up an equation that defines a function.

a. After reading the problem, draw a picture that conveys the given information.
b. State in your own words what the problem is asking for. Assuming that the problem asks you to find a particular quantity or formula, assign a variable to denote that key quantity.
c. Label any other quantities in your figure that appear relevant. Are there any equations relating these quantities?
d. Find an equation involving the variable you identified in step 2. Substitute to obtain an equation involving only the required variables.

This technique of setting up the equation in a symbolic manner will lead to solving the related maximum or minimum problem.

The following are sample materials extracted from class exercises and homework problems.

i. Finding the dimensions that maximizes an area.
Suppose that you have 600 meters with which to build two adjacent rectangular corrals. The two corrals are to share a common fence on one side. Find the dimensions so that the total enclosed area is as large as possible.

ii. A maximization problem involving revenue.
Suppose that the following demand function relates the selling price, p, of an item to the quantity sold by the following equation: $p = - 1/3\ x + 30$ where p is in dollars. For which value of x will the revenue be a maximum? Also, compute the corresponding unit price and maximum revenue.

5. **The course will not focus solely on computational skills.** ***What reasoning skills will be taught in the course?***

Throughout the course, students will learn rules, procedures, properties, and theorems to assist in the solving problems in the most efficient method. They will be using critical thinking skills to choose the particular rule, procedure, and other necessary concepts that are needed to analyze and solve the problem. They will learn to organize their wealth of information for a particular concept and identify the definitions, rules, and theorems that are important. They will also infer from the problem what may be true. Their experiences with proofs will help in formulating conjectures that lead to a method of solving the problem. They will learn to study and understand the important concepts and not focus solely on computational skills.

The following are examples. In the study of logarithms, the student will need to know the product, quotient and power rules before a given quantity can be written using sums and differences of simpler logarithmic expressions. In solving an equation where the unknown appears in the exponent, student needs to know the definitions and rules of logarithms before all the real number roots can be found by use of computation on a calculator. In finding the dimensions of a rectangle with the maximum area, the student needs to know the formulas for the perimeter and area of a rectangle and the procedure of setting up equations that define functions. The critical thinking skills needed in setting up the problem, gathering the important definitions, rules and theorems, formulating a procedure to solve the problem, and finally computing the answer will be the focus of the course.

6. **Instructors will build a bridge from theory to practice and show students how to traverse this bridge.** ***How will instructors help students make connections between theory and practice?***

Throughout the course, the elementary functions are introduced at length using algebraic, verbal, tabular, and graphical forms. A variety of applications in everyday life, in the sciences, in industry, and in business are included using real-life data. Once the concepts, theorems and properties are introduced, the instructor will show how the solution fits into the context of the application problem. For example, the study of quadratic functions includes maximum and minimum problems that can be solved in a definitive manner. The function will serve to describe or summarize a given situation in a way that is concise and revealing. The instructor will start by helping the students set up the function as follows: 1.) by reading the problem carefully and drawing a picture that conveys the given information; 2). State in your words what the problem is asking for; 3). Assign a variable to denote the key quantity. Label other quantities that are relevant. 4). Find the function that defines the problem.

After the study of linear functions, the graphs of many real-world quantities can be closely approximated by a straight-line graph. For example, after analyzing sales figures for a particular model of CD player, the accountant for College Sound Company has produced the graph relating the selling price and the number of units that can be sold each month at that price. Students will be asked to find an equation that relates the number of units sold to the selling price. They will use the equation to determine the number of units sold in a month and to determine the price to sell a given amount of units per month.

Students are then challenged to identify and solve application problems on their own after their exposure to these completely worked out examples. The following are sample materials extracted from class exercises and homework problems.

i. A rancher who wishes to fence off a rectangular area finds that the fencing in the east-west direction will require extra reinforcement due to strong prevailing winds.

Because of this, the cost of fencing in the east-west direction will be \$12 per (linear) year, as opposed to a cost of \$8 per yard of fencing in the north-south direction. Find the dimensions of the largest possible rectangular area that can be fenced for \$4,800.

ii. A power station is on one side of a river that is ½ mile wide and a factory is 6 miles downstream on the other side. It costs \$6 per foot to run power lines overland and \$8 per foot to run them underwater. Find the most economical path for the transmission line from the power station to the factory.

iii. The United States relative growth rate in the year 2000 is 0.6% per year. Compute the doubling time for the population assuming that this is an exponential growth model. Round the answer to the nearest whole number of years.

iv. Using a linear model for depreciation.
A factory owner buys a new machine for \$8,000. After 10 years, the machine has a salvage value of \$500.
(a). Assuming linear depreciation, find a formula for the value V(t) of the machine after t years, where $0 \leq t \leq 10$.
(b). Use the depreciation function determined in part (a) to find the value of the machine after seven years.

Sample Syllabus

LEEWARD COMMUNITY COLLEGE
Mathematics and Natural Sciences Division
Course Syllabus
Math 135(Pre-Calculus:Elementary Functions) (3.0 credits)

Instructor:	**Office Hours:**	**Office Location:**
Phone:	**E-mail:**	

Catalog Course Description: A functional approach to algebra which includes polynomial, exponential and logarithmic functions; higher degree equations; inequalities; sequences; binomial theorem; partial fractions. This course is recommended for students majoring in mathematics, sciences or engineering.

Co-requisites:	None
Prerequisites:	C or better in Math 103 or equivalent
Recommended Preparations:	None

Textbook and other Resources:

- David Cohen, A Custom Edition of Precalculus, Prepared Exclusively for the Leeward Community College Mathematics Discipline.

Student Learning Outcomes:

Upon successful completion of Math 135, a student should:
- be proficient in solving equations and inequalities, including those involving radical, exponential, and logarithmic expressions.
- be able to analyze, combine, and find the inverses of functions.
- be able to use properties to construct graphs of relations and functions in the Cartesian plane, including the conic sections.
- be proficient in analyzing the properties of polynomial functions and their graphs, including symmetry, intercepts, and zeros with their multiplicities.
- be proficient in analyzing the properties of rational functions and their graphs, including domain, range, asymptotes, and intercepts.
- be proficient in analyzing the properties of exponential and logarithmic functions and their graphs, including domain, range, and asymptotes.
- be able to model and solve various applications problems related to the studied relations and functions

Withdrawal Deadlines:

Other Important Dates:

Attendance: Students are expected to attend each class session on time. They are also expected to stay for the entire duration of the class. Anticipated as well as unexpected absences should be discussed with the instructor. The acceptance of an excuse for absence is at the discretion of the instructor. Students are expected to have their textbooks and note-taking equipment (writing instrument and paper) during each class session. Beyond being physically present in the room students are expected to be alert and engaged in the lecture or discussion. Students will be responsible for all material, discussion, and assignments covered during any missed class session(s). Students who miss class (for whatever reason) should arrange to obtain a set of class notes from a classmate, read through the covered sections and attempt the assigned problems. Students who miss a class will NOT be provided with a repeat of the missed lecture.

Grading Policy:

Course evaluation will be done through quizzes and examinations:

- In class quizzes are worth 5 points each. The best 10 scores will count toward the course grade.
- Take home quizzes are worth 10 points each. The best 10 scores will count toward the course grade.
- Midterms are worth 50 points each. There are 3 midterms. Exam dates will be announced in class.
- <u>Cumulative Final</u> is worth 100 points.

Out of the 400 total base points in the course, letter grades will be assigned on the following basis:

- A student who accumulates 352 or more points earns an A.
- A student who accumulates 310 to 351.5 points earns a B.
- A student who accumulates 268 to 309.5 points earns a C.
- A student who accumulates 226 to 267.5 points earns a D.
- A student who accumulates less than 226 points earns an F.

Students are expected to take quizzes and exams on the dates that are given. The format of the quizzes could be either take home or in class. The dates of the quizzes may or may not be announced in advance. A grade of zero, 0, will be given for any missed quiz. Take home quizzes are due <u>at the beginning</u> of the class period on the particular due day. Failure to turn in a take home quiz at the start of the class period will result in a grade of zero, 0. An in class quiz may be given at any time during a class period. No extra time will be given to students who are late for taking the quiz. There will be <u>absolutely NO</u> make up for any missed quiz.

A grade of zero, 0, will be given for any missed examination. If you know that there may be some problem or situation that may prevent you from taking any exam, please let the instructor know in advance. In order to make up a missed exam, a student who misses an exam due to an emergency <u>MUST</u>

submit "written" evidence detailing the emergency. The evidence must include a third-party name and contact information (phone number, email address, etc.) in the event verification of the emergency is needed. It is important to note that a make-up exam may NOT be identical or even comparable in format to the exam originally administered as scheduled. If a student misses an exam without providing acceptable evidence of an emergency, the student will receive a score of zero, 0, for the missed exam. If the instructor accepts the student's excuse, the student must take the make up exam, other than the final, within two days after the student's return. For example, if a student returns on Wednesday and is granted an opportunity to make up for the missed exam, then the student must take the make up exam no later than Friday. A student who completes every other course requirement but misses the final exam due to an emergency and submits acceptable evidence in a timely manner will be assigned a tentative grade of "I" (incomplete) and must take a make up exam during the next semester. Note that it is the student's responsibility to contact the instructor the following semester in order to arrange a time to make up the final exam before the deadline.

Students are expected to check UH Portal and their hawaii.edu email accounts regularly for possible announcements. Students are responsible for downloading take home quizzes and possibly other handouts from UH Portal. In order to access the files on UH Portal, please follow the instructions below:

- Login to your UH Portal account
- Click on My Tools tab
- Make sure that the course registered for the semester are displayed (You may choose the desired semester from the dropdown menu.)
- Click on your course link (Math 135)
- Click on Files from the options
- Select the desired document to download

Note: You need Adobe Reader to open the files.

Student with Disabilities Statement:

Leeward Community College abides by Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990, which stipulate that no student shall be denied the benefits of an education "solely by reason of a handicap." Students with documented disabilities who believe that they may need accommodations in this class are encouraged to contact the Coordinator of the KAKO'O 'IKE (KI) program as soon as possible to ensure that such accommodations are implemented in a timely fashion. The KI office is located in L-208, across from the elevator in the library building or call for information at 455-0421.

Reading and Suggested Problems

Answers to nearly all of these problems are printed in the back of your textbook. Step by step solutions are presented in the Student's Solutions Manual. You will NOT be submitting these problems. This listing is NOT an exhaustive list of everything that you will learn this semester. You are expected to read the textbook and study the examples presented in the textbook. The text is as follows: David Cohen, A Custom Edition of Precalculus, Prepared Exclusively for the Leeward Community College Mathematics Discipline by David Cohen, 2006.

The following chapters are covered: Chapter 1(Fundamentals),Chapter 2(Equations), Chapter 3(Functions), Chapter 4 (Polynomial and Rational Functions, Applications to Optimization), Chapter 5 (Exponential and Logarithmic Functions), Chapter 10 (Systems of Equations), Chapter 11 (The Conic Sections), and Chapter 12 (Roots of Polynomial Equations).

Suggested problems by sections:

Section	Suggested Problems
1.6	#1-36
1.7	#39-50,53-58
2.2	#13-76,85-88
2.3	#1-12,15-28
2.4	#1-60
3.1	#5-26,29-38,45-48
3,2	#23-30
3.3	#21-32
3.4	#1-24
3.5	#1-18
3.6	#1-15
4.2	#5-30
4.5	#1-23
4.6	#27-44
4.7	#1-34,39-49
5.1	3-32,39-48
5.2	#1-20
5.3	#1-40
5.4	#1-26,43-60
5.5	#31-42
5.6	#1-24
5.7	#1-40
10.6	#1-20
11.2	#1-20
11.4	#1-24
11.5	#1-24
12.1	#1-66,71-74
12.2	#21-56
12.3	#1-24,31-44
12.4	#1-14,31-44
12.6	#1-26, 41-44
12.7	#1-26