Leeward Community College

Foundations Course Designation Proposal

Written Communication *Symbolic Reasoning* *Global & Multicultural Perspectives*

The LCC Foundations Board invites LCC disciplines to propose that a course satisfy an AA Degree Foundations requirement. The Foundations Board will review all proposals to ensure that approved courses meet Foundations Hallmarks. If clarification is needed, a Board member will contact the Division chair. If the Foundations Board approves the proposal, all sections of the course will be designated as satisfying the requirement for five years. NOTE: A Foundations course cannot fulfill Diversification or Focus (WI or HAP) requirements.

Division Chairs should submit this form and accompanying materials directly to the Foundations Board Chair.

Deadlines: The Foundations Board will accept proposals at any time. However, to ensure a Fall semester designation and inclusion in the subsequently year's *Catalog,* or for a Spring designation, deadlines given in the Foundations Board calendar must be followed.

REQUESTED INFORMATION

1. *Course information*. Alpha ____MATH______ Course number_____140X________

2. *Foundations area requested*. Check one.
____Written Communication __X__Symbolic Reasoning ____Global & Multicultural Perspectives

3. *Official course description*. Submit a copy of the course description from the current *Catalog*. The course description must be consistent with the Hallmarks of the Foundations area.

4. *Syllabus*. Submit a master syllabus. If multiple instructors teach the course and use varying texts and/or assignments, include at least a couple representative syllabi.

5. *Assessment*. Provide a brief explanation of how those teaching the course will demonstrate in five years that this course has been meeting the Foundations Hallmarks.

6. *Application questions*. Provide the requested information for the Foundations area (see page 2).

Eric Matsuoka

Proposer Name	Proposer Signature	Date

Jennie Thompson

Discipline Coordinator Name	Discipline Coordinator Signature	Date

Janice Ito

Division Chair Name	Division Chair Signature	Date

Foundations Hallmarks & Application Questions

Request for Approval of
MATH 140X: Accelerated Pre-calculus
as an FS course

Leeward Community College
Fall 2010

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MATH 140X: Accelerated Pre-calculus (4)
AA/FS
4 hours of lecture and guided study per week.

Course Description

MATH 140X is designed to provide an accelerated path to Calculus to students who have a strong background in College Algebra. Topics include the essential pre-calculus skills needed for success in calculus: functions, with special attention to polynomial, rational, exponential, logarithmic, and trigonometric functions; plane trigonometry; polar coordinates; and conic sections. Credit may not be earned for both MATH 140 and MATH 140X.

Student Learning Outcomes

Upon successful completion of Math 140X, a student should:

- Evaluate and simplify algebraic and trigonometric expressions by applying appropriate formal rules or algorithms.
- Analyze and graph functions and equations involving algebra, trigonometry, and analytic geometry.
- Construct proofs using trigonometric identities.
- Apply theory from algebra, trigonometry and analytic geometry to model and solve various real world application problems.
- Select and correctly utilize precise mathematical language and symbols to effectively communicate procedures and results.

Changes

Not applicable

Assessment

Descriptions and samples of course materials that illustrate how the course meets the Foundations Hallmarks are provided on the following pages.

Formal assessment of the Symbolic Reasoning Hallmarks will be conducted according the schedule and expectations established by the Foundations Board. Reports of these assessments will be submitted to the Foundations Board.

Hallmark 1: Students will be exposed to the beauty, power, clarity and precision of formal systems.

In MATH 140X, students analyze polynomial, rational, exponential, logarithmic, and trigonometric functions by extending the formal tools introduced in College Algebra (MATH 103). Through this analysis, students see how commonly encountered notions such as "exponential growth" and "seasonal variation" can be made clear and precise using formal mathematical models. Students also see how seemingly unrelated phenomena, such as the distance an object falls due to gravity and the kinetic energy of an object (such as a car) as its speed varies, share the same formal mathematical model. Students will realize the power in being able to model and study familiar situations and the beauty of seeing the sometimes-unexpected relationships between these models.

Sample course materials:

- (Take-home assignment) Suppose the doubling time for the earth's population is 60 years. If the earth's population was 1.6 billion people in the year 1900, construct an exponential model for the earth's population P(t) as a function of the year t. Use this model to predict the earth's population in the years 1800 and 2000 and compare these predictions to the actual populations of 959 million and 6.1 billion respectively. Use the model you constructed to predict the year in which the earth's population will reach 10 billion.
- (Class discussion) Discuss how the lay meaning of "seasonal variation" compares to the formal definition of a periodic function: f(x) is periodic with period p if there exists a constant p for which f(x + p) = f(x) for all x in the domain of f. Discuss how sine and cosine functions, their compositions, and their combinations might be used to model periodic functions then show the report *Studying seasonality by using sine and cosine functions in regression analysis* to show how the modeling can and has been done. The report is available from the National Institute of Health web site: http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1756865/pdf/v053p00235.pdf
- (Take-home assignment) Search through some authoritative source to find the classical mechanics physics formula for kinetic energy and the distance an object falls due to the earth's gravity (ignoring wind resistance) and use the formulas to answer the following questions. [Students will find the formulas $E = \frac{1}{2}mv^2$ and $s = 4.9t^2$ (in meters per second)]
    - If an object has constant mass (say 1 kilogram), what is the effect on kinetic energy of doubling the velocity? What is the effect on kinetic energy of tripling the velocity?
    - If an object is falling on earth (neglecting wind resistance), what is the effect on the distance the object falls for twice the time? What is the effect on the distance the object falls if the object falls for triple the time?
    - If mass is held constant, explain why the formulas represent identical mathematical models (units and nonzero constant multiples aside). (hint: what is E directly proportional to and what is s directly proportional to?)

Hallmark 2: Instructors will help students understand the concept of proof as a chain of inferences.

Proofs of important theorems, properties, and formulas are either shown or at least discussed in class. Students see the importance of comparing given and assumed information to hypotheses of a theorem in order to logically deduce the conclusion. Counterexamples illustrate how conclusions cannot be assumed if the hypotheses are not met.

Sample course materials:

- (Class presentation) The Fundamental Theorem of Algebra (FTA) states that an n^{th} degree polynomial function has exactly n complex zeros (where we count a zero of multiplicity m as m zeros). The proof of this theorem relies on a seemingly-weaker form: Every non-constant polynomial function has a complex zero. To prove this seemingly-weaker form, we need Liouville's Theorem from complex analysis: every bounded entire function must be a constant function. Using a proof by contradiction, suppose P(x) is a non-constant polynomial function that has no complex zero. We consider the function $f(x) = \frac{1}{P(x)}$ which is a rational function where the denominator is never zero. A rational function where the denominator is never zero is entire and bounded so by Liouville's Theorem, f(x) must be a constant function. If f(x) is a constant function then P(x) must also be a constant function but this contradicts our premise that P(x) is a non-constant polynomial function. This contradiction shows that a non-constant polynomial function must have a complex zero. So, how do we get from this seemingly-weaker form to the original form? That comes from the Factor Theorem that we proved in class: if P(x) is a polynomial function then P(a) = 0 if and only if $P(x) = (x-a)Q(x)$ for some polynomial function Q(x). Applying the combination of the weaker version of the FTA and factor theorem n times to an n^{th} degree polynomial P(x) gives: $P(x) = (x-a_1)(x-a_2)...(x-a_n)Q(x)$. Elementary polynomial multiplication shows that this Q(x) must be a constant so P(x) has exactly n zeros.
- (Class discussion) $f(x) = e^{x^2-1}$ has no real or complex zeros. Why is it that this function does NOT disprove the second form of the FTA?
- (Take-home assignment) In class we proved that $\log_b(uv) = \log_b u + \log_b v$ provided each logarithm is defined. Use a similar technique to prove that $\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$ provided each logarithm is defined.
- (Homework) Prove the trigonometric identity: $\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

Hallmark 3. Instructors will teach students how to apply formal rules or algorithms.

There are many algorithms that are introduced in MATH 140X. Instructors will carefully teach students how to apply them so that their results can be used to solve problems. Some examples of these include:

- To use the unit circle to evaluate a trigonometric function at a given value θ (where θ is a multiple of $\frac{\pi}{6}$ or $\frac{\pi}{4}$,
  1. Determine the quadrant in which the angle θ intersects the unit circle.
  2. From the quadrant, determine the signs of $\cos\theta$ and $\sin\theta$.
  3. Determine the correct reference angle associated with θ.
  4. Evaluate the trigonometric function using the results of 2, 3, and (if necessary) the trigonometric relationships $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, and $\csc\theta = \frac{1}{\sin\theta}$.
- To sketch the graph of a rational function (in lowest terms) $f(x) = \frac{N(x)}{D(x)}$,
  1. Find the zeros and their multiplicities of N(x). These are the x-intercepts of the graph. As a reminder, f(x) has a sign change at odd multiplicity zeros of N(x) and no sign change at even multiplicity zeros.
  2. Find the zeros and their multiplicities of D(x). These are the vertical asymptotes of the graph. As a reminder, f(x) has a sign change at odd multiplicity zeros of D(x) and no sign change at even multiplicity zeros of D(x).
  3. Use long division to re-write f(x) in the form $f(x) = q(x) + \frac{r(x)}{D(x)}$.
     - If q(x) is a constant c then y = c is a horizontal asymptote of f(x).
     - If q(x) is a linear function mx + b then the line y = mx + b is a slant asymptote of f(x).
     - If q(x) has degree two or greater then q(x) has no horizontal or slant asymptote.
     - If q(x) has a horizontal or slant asymptote then zeros of r(x) are the x values where the graph crosses the horizontal or slant asymptote.
  4. Evaluate f(0) to determine the y-intercept of the graph (provided 0 is in the domain of f).
  5. Sketch the graph of f(x) using the information gathered in #1 to #4.

Hallmark 4. Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence.

After analyzing a problem and determining that an algebraic or trigonometric technique is applicable, students will construct an appropriate symbolic model; manipulate the model using algebraic and/or trigonometric algorithms taught in the course; interpret the model and the manipulations in order to draw conclusions; check the validity of the conclusion; and (when required) explain or defend the conclusions by pointing out the appropriateness of the model they constructed and manipulations they performed.

The best illustration of this process is in the solving of verbal problems. The general procedure is:

1. Read the problem to determine what is given, what must be assumed, what is asked for, and what information is extraneous.
2. Based on the analysis in step 1, model the problem through some combination of the following:
   - Define variables to represent desired quantities and other unknowns in symbolic form.
   - Draw an appropriate diagram and label the diagram with given information and desired variables.
   - Find formula(s) that is/are applicable to the problem. Identify the given and desired values of the variables in the each formula. Determine if other variables are truly unknown or if they can be solved for in terms of the given or desired values.
   - Determine if theorems or properties are applicable to the given information in the problem.
   - Use given information, theorems, and/or properties to construct graphs, equations, inequalities, and/or systems that represent given, proven, or known relationships between the known and desired quantities.
3. Apply algebraic, or trigonometric techniques to the symbolic representations to solve for the desired quantity/quantities.
4. Check the validity of all answers and be prepared to explain why the modeling and/or techniques is/are appropriate and lead to the answer(s) obtained.

Some course-appropriate example problems that use this symbolic problem solving process and could be used as examples in class or assigned as homework, a quiz problem, or an exam problem are:

- A flexible room divider 7 feet tall and 20 feet long will be used to partition a rectangular storage area in the corner of an office. What should the dimensions of the rectangle be in order to maximize the area of the storage area?
- A surveyor 5' 6" tall is 200 feet from the base of a building. The surveyor's inclinometer (used to measure angles of inclination) is mounted on a tripod 5' above the ground. The inclinometer shows that the angle of inclination to the top of the building is 42 degrees. How tall is the building?
- Suppose a weight is attached to a coiled spring. The weight is pulled 4 inches from its equilibrium position then released. The time for one complete oscillation is 3 seconds. Find a symbolic formula that models the position of the weight relative to the equilibrium position at time t seconds, determine the position of the weight at time 2 seconds, and analyze the formula to determine the frequency.

Hallmark 5. The course will not focus solely on computational skills.

In MATH 140X, students are required to perform many different kinds of computations but such computations are not the focus of the course.

Some computations are used to deduce or illustrate relationships between variables.

- In class, the Law of Cosines formula $c^2 = a^2 + b^2 - 2ab\cos C$ is proven symbolically. Instructors make a point to inform students that the Law of Cosines is a generalization of the Pythagorean Theorem to non-right triangles. Students then evaluate the formula for fixed side measurements *a* and *b* and varying included angle measurements *C* then use the results of these calculations to make associations between the calculated side measurement *c* and the included angle measurement *C*.

Computations often provide the intermediate steps needed to prove a theorem, formula, or property.

- The Law of Cosines formula $c^2 = a^2 + b^2 - 2ab\cos C$, the triangular area formula $K = \frac{1}{2}ab\sin C$, and the Pythagorean Identity $\cos^2 C + \sin^2 C = 1$ combine with substantial algebraic computations to prove Heron's beautiful formula for triangular area $K = \sqrt{s(s-a)(s-b)(s-c)}$.

In other cases, computations are necessary to re-write a formula in a form that can be properly studied.

- Using long division computations to re-write the formula for the rational function $f(x) = \frac{x^2 - 5x + 1}{x - 2}$ as $f(x) = x - 3 + \frac{-5}{x-2}$ allows us to conclude that:
    1. Since $\frac{-5}{x-2} \to 0$ as $x \to \pm\infty$, the graph of $f(x)$ has the line $y = x - 3$ as a slant asymptote.
    2. Since $\frac{-5}{x-2} > 0$ as $x \to -\infty$, the graph of $f(x)$ approaches its slant asymptote from above as $x \to -\infty$.
    3. Since $\frac{-5}{x-2} < 0$ as $x \to \infty$, the graph of $f(x)$ approaches its slant asymptote from below as $x \to \infty$.
    4. Since $\frac{-5}{x-2} \neq 0$ for all x, the graph of $f(x)$ never crosses its slant asymptote.

Hallmark 6. Instructors will build a bridge from theory to practice and show students how to traverse this bridge.

Applications of the MATH 140X concepts and techniques will be emphasized in the course. Instructors will demonstrate the practical applications of the theory and assign homework and quiz and exam problems that require students to similarly apply the new material.

Sample course material includes:

- After introducing exponential growth, students are shown applications that include:
  1. Instructors establish that the exponential function $P(t) = P_0 e^{rt}$ is used to model continuously compounded interest then show that this "nice" formula is an excellent approximation for daily compounded interest, whose formula is much less pleasant to work with: $P(t) = P_0\left(1+\frac{r}{365}\right)^{365t}$. Students would be assigned to contact their favorite local savings institution, ask for their current annual interest rate for savings accounts, and calculate the future value based on a hypothetical initial deposit. They would then be referred to the web site www.money-rates.com/savings.htm (or similar page) to see some of the best savings interest rates nation-wide. Students would calculate the future value of the same hypothetical investment using the best rate listed there to see practical uses of the formula and actual dollar differences between local interest rates and some of the best-available national rates.
  2. Using the same model as in the above example, students would be shown that the time it takes for a continuously-compounded interest investment to double is independent of the initial deposit amount. They would then be instructed to calculate and compare the doubling times for the local savings interest rate and the (usually) better national savings interest rate. This provides students with another practical way of viewing the difference between interest rates.
  3. The earth's population is another practical example of exponential growth. After using historical data to establish a reasonable model $P(t) = P_0 e^{rt}$, students will be instructed to research estimates of the earth's carrying capacity and to use the model to predict when the earth will reach its carrying capacity based on the estimates that various experts have opined.
- Sample course material involving trigonometry include:
  1. An aerial photograph from a U-2 spy plane is taken of a building suspected of housing nuclear warheads. The photograph is made when the angle of elevation of the sun is 32 degrees. By comparing the shadow cast by the building to objects of known size in the photograph, analysis determines that the shadow is 80 feet long. How tall is the building?
  2. For the past three years, a retail store manager observed that the revenue reaches of an annual high of about $40,000 in December and a low of about $10,000 in June with a graph that looks like a sinusoid. If we begin numbering the months of the year starting with "1" for January and fit this data into a sinusoidal model, what are the period, amplitude, phase shift, vertical translation, and formula? Use the model to predict the revenue each October.

Leeward Community College
Mathematics & Natural Sciences Division
MATH 140X: Accelerated Pre-Calculus (4 credits)
Sample Course Syllabus

Instructor:
Contact information:
Office location and hours:

Catalog course description: MATH 140X is designed to provide an accelerated path to Calculus to students who have a strong background in College Algebra. Topics include the essential pre-calculus skills needed for success in calculus: functions, with special attention to polynomial, rational, exponential, logarithmic, and trigonometric functions; plane trigonometry; polar coordinates; and conic sections. Credit may not be earned for both MATH 140 and MATH 140X.

Prerequisite: Any one of the following, or an articulated equivalent, within the past two years, will qualify a student for MATH 140X: A in MATH 103 OR C in MATH 135 OR qualified placement test score (62 or higher in the COMPASS college algebra placement domain) OR "well prepared" designation in the Algebra II End-Of-Course exam OR instructor consent.

Co-requisites: None

Recommended preparation: None

STUDENT LEARNING OUTCOMES

Upon successful completion of Math 140X, a student should:
- Evaluate and simplify algebraic and trigonometric expressions by applying appropriate formal rules or algorithms.
- Analyze and graph functions and equations involving algebra, trigonometry, and analytic geometry.
- Construct proofs using trigonometric identities.
- Apply theory from algebra, trigonometry and analytic geometry to model and solve various real world application problems.
- Select and correctly utilize precise mathematical language and symbols to effectively communicate procedures and results.

REQUIRED COURSE MATERIALS

- Required textbook bundle (ISBN #0558928552):
  - Fundamentals of Precalculus, Second Edition, by Mark Dugopolski
  - Video Lectures on CD to accompany Fundamentals of Precalculus, Second Edition
  - MyMathLab Student Access Kit
  - A Review of Algebra, by Heidi Howard
- Hand-held scientific calculator
- Regular Internet access

GRADING POLICY

C or better eligibility: To be eligible for a C or better letter grade (or equivalently a CR grade for the CR/NC grading option), a student must score at least 85% on a departmental exit assessment that can be taken on campus or at home. The exit assessment can be repeated until as often as is needed to score at least 85%. A student who does not score at least 85% by Thursday of Finals Week cannot earn a C or better for the course. IMPORTANT: Scoring at least 85% on the departmental exit assessment makes a student eligible to earn a C or better (or CR) for MATH 140X but does not by itself guarantee a C or better letter grade.

Course letter grades are determined by the following components:

- Computerized homework
- Paper/pencil quizzes
- Attendance
- Exams

Subject to the "C or better eligibility" requirement listed above,

- A student whose percent average for every component is at least 90% earns an A.
- A student whose percent average for every component is at least 80% earns a B.
- A student whose percent average for every component is at least 70% earns a C (or CR).
- A student whose percent average for every component is at least 60% earns a D.
- A student who does not earn a D or better earns an F.

Computerized homework: There are a total of 50 computerized homework assignments covering every chapter of the textbook. Each problem allows multiple attempts to find the correct answer and earn full credit. Each assignment has a due date that must be met in order to earn full credit. A scoring penalty of 20% will be assessed for additional work that is done after the due date.

Paper/pencil quizzes: There are 4 paper/pencil quizzes that cover the following topics:

1. Polynomial and rational functions
2. Trigonometric equations and identities
3. Exponential and logarithmic functions
4. Conic sections

The quizzes must be taken on campus during class time or in the campus (paper) testing center in room BE-227. The quizzes are repeatable but are limited to one per day. Quiz scoring is based on both the correctness of the work and on the presentation of the work (notation, terminology, use of symbols, etc.). A first attempt at a quiz can be made any time prior to the quiz due date. A student is able to re-take a quiz to improve their score after satisfactorily completing a paper/pencil worksheet based on similar problems. Each quiz may be attempted only once on any given day.

Attendance: Each scheduled class meeting consists of two sessions. The first session, which includes the first 30 minutes of class, will consist of presentation of new topics, procedures, and examples. For the second session, students will spend the remaining 90 minutes of class time in guided semi-independent study working through the computerized assignment.

A student who either completes the day's homework assignment(s) prior to class time, or is present for at least 20 minutes of the first session of the class, will earn 2 points. Students who spend at least 60 of the 90 minute second class session working on a paper/pencil quiz or actively doing the computerized “homework” assignments will earn 1 point. A student’s percent average is found by dividing the student’s earned attendance points by the 90 possible attendance points (3 possible points per day multiplied by 30 scheduled class days).

Exams: There are 5 computerized chapter exams. Each exam has a 90 minute time limit. A first attempt at a chapter exam can made any time prior to the exam due date. A student can re-take an exam to improve their score after satisfactorily completing a computerized chapter review problem set. Each exam may be attempted only once on any given day.

STUDENT ASSESSMENT NOTIFICATION

With the goal of continuing to improve the quality of educational services offered to students, Leeward CC conducts assessments of student achievement of course, program, and institutional learning outcomes. Student work is used anonymously as the basis of these assessments, and the work you do in this course may be used in these assessment efforts.

STUDENT WITH DISABILITIES STATEMENT

Leeward Community College abides by Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990, which stipulate that no student shall be denied the benefits of an education "solely by reason of a handicap." Students with documented disabilities who believe that they may need accommodations in this class are encouraged to contact the Coordinator of the KAKO‘O ‘IKE (KI) program as soon as possible to ensure that such accommodations are implemented in a timely fashion. The KI office is located in L-208, across from the elevator in the library building or call for information at 455-0421.